

DSF 26/6/26

$$1) p = 0.55 \quad x = \frac{0.45}{0.55} = \frac{9}{11}$$

$$P_6 = \frac{\left(\frac{9}{11}\right)^6 - \left(\frac{9}{11}\right)^{20}}{1 - \left(\frac{9}{11}\right)^{20}} = 28,7\%$$

$$\frac{1}{2}: \quad P_6 = 1 - \frac{6}{20} = \frac{7}{10} = 70\%$$

$$0.25 = 1 - \frac{6}{N} = \frac{N-6}{N}$$

$$\frac{1}{4} N = N - 6$$

$$\frac{3}{4} N = 6$$

$$\boxed{N=8}$$

$$2) \quad x_{h+2} + 3x_{h+1} + \alpha x_h = 3$$

$$(4+\alpha)\bar{x} = 3$$

$$\bar{x} = \frac{3}{4+\alpha}$$

$$\alpha \neq -4$$

$$\lambda^2 + 3\lambda + \alpha$$

$$3 < \alpha + 1 < 2$$

UNST.

$$\alpha = 2$$

$$\lambda^2 + 3\lambda + 2 = 0$$

$$\bar{x} = \frac{1}{2}$$

$$\frac{-3 \pm \sqrt{9-8}}{2} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$$

$$x_h = c_1 (-1)^h + c_2 (-2)^h + \frac{1}{2}$$

$$\begin{cases} c_1 + c_2 + \frac{1}{2} = 0 \\ -c_1 - 2c_2 + \frac{1}{2} = 1 \end{cases}$$

$$c_1 = -c_2 - \frac{1}{2}$$

$$c_2 + \frac{1}{2} - 2c_2 + \frac{1}{2} = 1$$

$$c_2 = 0 \quad c_1 = -\frac{1}{2}$$

$$x_h = -\frac{1}{2}(-1)^h + \frac{1}{2} \quad 0, 1, 0, 1, 0, 1, \dots$$

$$3) \quad P=0 \quad \begin{cases} S=0 \\ S=2 \end{cases} \quad S=0 \quad P=\begin{cases} 0 \\ 3 \end{cases}$$

$$\begin{cases} P+2S=3 \\ P+S=2 \end{cases} \quad \begin{matrix} S=1 \\ P=1 \end{matrix}$$

$$(0,0) \quad (1,1) \quad (0,2) \quad (3,0)$$

$$\nabla F = \begin{bmatrix} 3-2P-2S & -2P \\ -S & 2-P-2S \end{bmatrix}$$

$$\nabla F(0,0) = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \quad \text{UNST. NODE}$$

$$\nabla F(1,1) = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$$

$$\tau = -2 \quad \delta = -1 \quad \text{SADDLE UNST.}$$

$$\nabla F(0,2) = \begin{bmatrix} -1 & 0 \\ -2 & -2 \end{bmatrix} \quad \text{A.S.T. (NODE)}$$

$$\nabla F(3,0) = \begin{bmatrix} -3 & -6 \\ 0 & -1 \end{bmatrix} \text{ A.S. (NODE)}$$

$$\begin{aligned} 4) \quad \ddot{G} &= -2\dot{G} - \gamma - k\dot{\gamma} \\ &= -2\dot{G} - \gamma - k(-3\gamma + G + 2) \\ &= -2\dot{G} - \gamma + 3k\gamma - kG - 2k \end{aligned}$$

$$\begin{cases} \dot{\gamma} = -3\gamma + G + 2 \\ \dot{G} = z \\ \dot{z} = (3k-1)\gamma - kG - 2z - 2k \end{cases}$$

$$\begin{cases} G = 3\gamma - 2 \\ z = 0 \\ (3k-1)\gamma - kG = 2k \end{cases}$$

$$(\cancel{3k}-1)\gamma - \cancel{3k}\gamma + \cancel{2k} = \cancel{2k} \quad \gamma = 0$$

$$(0, -2, 0)$$

$$\det \begin{bmatrix} -3-\lambda & 1 & 0 \\ 0 & 0-\lambda & 1 \\ 3k-1 & -k & -2-\lambda \end{bmatrix} = \lambda(\lambda^2 + 5\lambda + 6) - \lambda(\lambda+2)(\lambda+3) + 3k-1 - k(3+\lambda)$$

$$\lambda^3 + 5\lambda^2 + 6\lambda + 1 - \cancel{3k} + \cancel{3k} + k\lambda$$

$$\lambda^3 + 5\lambda^2 + (6+k)\lambda + 1$$

coeff. positive

$$H = \begin{bmatrix} \boxed{\begin{matrix} 5 & 1 \\ 1 & 6+k \end{matrix}} & 0 \\ 0 & 5 & 1 \end{bmatrix}$$

$$5(6+k) > 1$$

$$k > \frac{1}{5} - 6 \quad \forall k > 0$$

G.A.S.

$$5) \quad x(1+\alpha-x^2) = x$$

$$\bar{x} = 0 \quad \cancel{1+\alpha-x^2=x} \quad x^2 = \alpha$$

$$\bar{x} = \pm \sqrt{\alpha} \quad \alpha > 0$$

$$f'(x) = 1 + \alpha - 3x^2$$

$$f'(0) = 1 + \alpha$$

$$-1 < 1 + \alpha < 1$$

$$-2 < \alpha < 0 \quad \bar{x} = 0$$

A.S.

$$\alpha > 0$$

$$\alpha < -2 \vee \alpha > 0 \quad \bar{x} = 0 \text{ UNST.}$$

$$f'(\pm\sqrt{\alpha}) = 1 + \alpha - 3\alpha = 1 - 2\alpha$$

$$-1 < 2\alpha - 1 < 1 \quad 0 < 2\alpha < 2$$

$$0 < \alpha < 1 \quad \bar{x} = \pm\sqrt{\alpha} \text{ A.S.}$$

$$\alpha > 1 \quad \text{UNST.}$$

