

$$17/7/26$$

$$1) \quad q = 0.52 \quad x = \frac{0.48}{0.52} = \frac{12}{13}$$

$$p_5 = \frac{x^5 - x^{18}}{1 - x^{18}} = 0.568 = 56.8\%$$

$$q = \frac{1}{2} \quad p = 72.2\%$$

$$1 - \frac{5}{N} = \frac{1}{2} \quad \frac{5}{N} = \frac{1}{2} \quad N = 10$$

$$2) \quad x_{h+2} - x_{h+1} + \alpha x_h = 2$$

$$2\bar{x} = 2 \quad \bar{x} = \frac{2}{2} \quad 2 \neq 0$$

$$\lambda^2 - \lambda + \alpha$$

$$1 < 1 + \alpha < 2$$

$$0 < \alpha < 1 \quad \text{A.S.}$$

$$\alpha < 0 \vee \alpha > 1 \quad \text{inst.}$$

$$2 = -2 \quad \bar{x} = -1$$

$$\lambda^2 - \lambda - 2 = 0$$

$$\frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2} \quad \begin{matrix} -1 \\ 2 \end{matrix}$$

$$x_h = c_1 (-1)^h + c_2 2^h - 1$$

$$x_0 = x_1 = 1$$

$$\begin{cases} c_1 + c_2 - 1 = 1 \\ -c_1 + 2c_2 - 1 = 1 \end{cases}$$

$$C_2 = 2 - C_1$$

$$-C_1 + 4 - 2C_1 = 2 \quad + 3C_1 = +2 \quad C_1 = \frac{2}{3}$$

$$C_2 = 2 - \frac{2}{3} = \frac{4}{3}$$

$$x_k = \frac{2}{3} (-1)^k + \frac{4}{3} 2^k - 1$$

$$3) \begin{cases} P(4 - P - 2S) = 0 \\ S(3 - 2P - S) = 0 \end{cases}$$

$$P=0 \quad S=0$$

$$P=0 \quad S=3$$

$$S=0 \quad P=4$$

$$\begin{cases} P + 2S = 4 \\ 2P + S = 3 \end{cases}$$

$$P = 4 - 2S$$

$$8 - 4S + S = 3$$

$$3S = 5 \quad S = \frac{5}{3}$$

$$(0, 0) \quad (0, 3) \quad (4, 0) \quad \left(\frac{2}{3}, \frac{5}{3}\right)$$

$$P = 4 - \frac{10}{3} = \frac{2}{3}$$

$$A = \begin{bmatrix} 4 - 2P - 2S & -2P \\ -2S & 3 - 2P - 2S \end{bmatrix}$$

$$A(0, 0) = \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix} \quad \text{UNST. NODE}$$

$$A(0, 3) = \begin{bmatrix} -2 & 0 \\ -6 & -3 \end{bmatrix} \quad \text{ST. NODE}$$

$$A(4, 0) = \begin{bmatrix} -4 & -8 \\ 0 & -5 \end{bmatrix} \quad \text{ST. NODE}$$

$$A\left(\frac{2}{3}, \frac{5}{3}\right) = \begin{bmatrix} -\frac{2}{3} & -\frac{4}{3} \\ -\frac{10}{3} & -\frac{5}{3} \end{bmatrix} \quad \begin{aligned} \Delta &= -\frac{7}{3} < 0 \\ \delta &= \frac{10}{9} - \frac{40}{9} = -\frac{10}{3} < 0 \end{aligned}$$

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$$4) \begin{cases} \dot{Y} = -Y + G + 5 \\ \dot{G} = -3G - \int_0^t Y(\tau) d\tau + kY \end{cases} \quad k > 0$$

$$Z = \dot{G}$$

$$\begin{aligned} \dot{Z} &= -3Z - Y + k(-Y + G + 5) \\ &= -3Z - Y - kY + kG + 5k \end{aligned}$$

$$\begin{cases} \dot{Y} = -Y + G + 5 \\ \dot{G} = Z \\ \dot{Z} = -(1+k)Y + kG - 3Z + 5k \end{cases}$$

$$\bar{Z} = 0$$

$$\bar{Y} = \bar{G} + 5$$

$$-\bar{G} - 5 - k\bar{G} - 5k + k\bar{G} + 5k = 0$$

$$\bar{G} = -5$$

$$(0, -5, 0)$$

$$A = \begin{bmatrix} -1-\lambda & 1 & 0 \\ 0 & 0-\lambda & 1 \\ -(1+k) & k & -3-\lambda \end{bmatrix}$$

$$-(\lambda^2 + \lambda)(3 + \lambda) - (1 + k) + k(\lambda + 1) \\ + 3\lambda^2 + 3\lambda + \lambda^3 + \lambda^2 + 1 + \cancel{k} - k\lambda - \cancel{k}$$

$$\lambda^3 + 4\lambda^2 + (3-k)\lambda + 1 \quad k < 3$$

$$\left[\begin{array}{cc|c} 4 & 1 & 0 \\ 1 & 3-k & 0 \\ \hline 0 & 4 & 1 \end{array} \right] \quad \begin{array}{l} 4(3-k) > 1 \\ 3-k > \frac{1}{4} \end{array} \quad \boxed{k < \frac{11}{4}} \quad \text{A.S.}$$

$$5) \quad x_{n+1} = x_n(1 + \alpha - 2x_n^2)$$

$$x(1 + \alpha - 2x^2) = x$$

$$x = 0$$

$$1 + \alpha - 2x^2 = 1$$

$$x = \pm \sqrt{\frac{\alpha}{2}} \quad \alpha > 0$$

$$f'(x) = 1 + \alpha - 6x^2$$

$$f'(0) = 1 + \alpha$$

$$-1 < 1 + \alpha < 1$$

$$-2 < \alpha < 0 \quad \text{A.S.}$$

$$\alpha < -2 \cup \alpha > 0 \quad \text{UNST.}$$

$$f'(\pm\sqrt{\frac{\alpha}{2}}) = 1 + \alpha - 3\alpha = 1 - 2\alpha$$

$$\alpha > 0$$

$$-1 < 2\alpha - 1 < 1$$

$$0 < 2\alpha < 2$$

$$0 < \alpha < 1 \quad \text{A.S.} \quad 0$$

$$\alpha \geq 1 \quad \text{UNST.}$$

