

UNIVERSITÀ CATTOLICA DEL SACRO CUORE

Dynamical Systems in Finance **17th July 2026**

1. A gambler starts from 5€ and aims to reach a total of 18€. If the probability of winning is 0.52, what is the probability that the gambler gets ruined?

What happens if the probability of winning lowers to 1/2? In this case, what would be the target N needed to have exactly a 50% chance of getting ruined?

2. Discuss the stability of equilibria of the linear second-order discrete equation

$$x_{h+2} - x_{h+1} + \alpha x_h = 2$$

for $\alpha \in \mathbb{R}$.

In the case $\alpha = -2$, find the explicit solution for $x_0 = 1$, $x_1 = 1$.

3. The interaction between the price and the supply of a good follows the system of ODEs

$$\begin{cases} \dot{P} = P(4 - P - 2S) \\ \dot{S} = S(3 - 2P - S). \end{cases}$$

Discuss the stability of all the equilibria. Are there any saddles, nodes or foci?

4. A Phillips' stabilization model with a mixed integral-proportional policy is of the form

$$\begin{cases} \dot{Y} = -Y + G + 5 \\ \dot{G} = -3G - \int_0^t Y(\tau) d\tau + kY \end{cases}$$

for $k > 0$.

Write the equilibrium position and discuss its stability.

5. Discuss the stability of the equilibria of the d.d.e.

$$x_{h+1} = x_h(1 + \alpha - 2x_h^2), \quad \alpha \in \mathbb{R}.$$