

DSF

26.11.2026

$$1) \quad x = \frac{0.45}{0.55} = \frac{9}{11}$$

$$p_5 = \frac{x^5 - x^N}{1 - x^N} = \frac{\left(\frac{9}{11}\right)^5 - \left(\frac{9}{11}\right)^{15}}{1 - \left(\frac{9}{11}\right)^{15}} = 33.4\%$$

Fair: $p_5 = 1 - \frac{5}{15} = 1 - \frac{1}{3} = \frac{2}{3} = 66.7\%$

$$1 - \frac{5}{N} = \frac{3}{4} \quad \frac{5}{N} = \frac{1}{4} \quad \frac{N}{5} = 4 \quad N = 20$$

$$2) \quad \bar{x} - 4\bar{x} - 5\bar{x} + 8 = 0 \quad \bar{x} = 1$$

$$\lambda^2 - 4\lambda - 5 = 0$$

$$4 < 1 - 5 < 2 \quad \text{UNST.}$$

$$\lambda_{1/2} = 2 \pm \sqrt{4+5} = 2 \pm 3 < \frac{-1}{5}$$

$$x_h = c_1(-1)^h + c_2 5^h + 1$$

$$x_0 = c_1 + c_2 + 1 = 2$$

$$x_1 = -c_1 + 5c_2 + 1 = 6$$

$$2 + 6c_2 = 8 \quad c_2 = 1$$

$$c_1 = 1 - c_2 = 0$$

$$x_h = 1 + 5^h$$

$$3) \quad \begin{cases} x^2 y - x = 0 & x^3 - x = 0 & x(x^2 - 1) = 0 \\ \cancel{x(x-y)} = 0 & x = y \end{cases}$$

$$x = 0 \quad y = 0$$

$$x = \pm 1 \quad y = \pm 1$$

$$(0, 0) \quad (1, 1) \quad (-1, -1)$$

$$\nabla F = \begin{bmatrix} 2xy - 1 & x^2 \\ \alpha & -\alpha \end{bmatrix}$$

$$\nabla F(0, 0) = \begin{bmatrix} -1 & 0 \\ \alpha & -\alpha \end{bmatrix} \quad \begin{array}{ll} \alpha > 0 & \text{ST. NODE} \\ \alpha < 0 & \text{SADDLE} \end{array}$$

$$\nabla F(1, 1) = \begin{bmatrix} 1 & 1 \\ \alpha & -\alpha \end{bmatrix} \quad \begin{array}{l} \tau = 1 - \alpha \\ \delta = -2\alpha \end{array}$$

$$\alpha > 0 \quad \text{SADDLE}$$

$$\alpha < 0 \quad \tau > 0 \quad \begin{aligned} \tau^2 - 4\delta &= 1 + \alpha^2 - 2\alpha + 8\alpha \\ &= 1 + \alpha^2 + 6\alpha > 0 \end{aligned}$$

$$\alpha_{1/2} = -3 \pm \sqrt{9 - 1} = -3 \pm \sqrt{8}$$

$$\begin{array}{ll} \alpha \leq -3 - \sqrt{8} \quad \vee \quad -3 + \sqrt{8} \leq \alpha < 0 & \text{UNST. NODE} \\ -3 - \sqrt{8} < \alpha < -3 + \sqrt{8} & \text{UNST. FOCUS} \end{array}$$

$$(-1, -1) \quad \text{same as } (1, 1)$$

$$4) \begin{cases} \dot{Y} = -2Y + 4G - 8 \\ \dot{G} = Z \\ \dot{Z} = -2Y - kY - 3Z \\ \quad = 4Y - 8G + 16 - kY - 3Z \end{cases}$$

$$\begin{cases} \dot{Y} = -2Y + 4G - 8 \\ \dot{G} = Z \\ \dot{Z} = (4-k)Y - 8G - 3Z + 16 \end{cases}$$

$$\bar{Z} = 0$$

$$-2\bar{Y} + 4\bar{G} = 8$$

$$(4-k)\bar{Y} - 8\bar{G} = -16$$

$$(\cancel{-4} + \cancel{4} - k)\bar{Y} = 0$$

$$\bar{Y} = 0$$

$$\bar{G} = 2$$

$$(0, 2, 0)$$

$$A = \begin{bmatrix} -2-\lambda & 4 & 0 \\ 0 & 0-\lambda & 1 \\ 4-k & -8 & -3-\lambda \end{bmatrix}$$

$$\begin{aligned} \delta &= \lambda(\lambda+2)(-3-\lambda) + 4(4-k) - 8(2+\lambda) \\ &= (\lambda^2+2\lambda)(-3-\lambda) + \cancel{16} - 4k - \cancel{16} - 8\lambda \\ &= -\lambda^3 - 3\lambda^2 - 2\lambda^2 - 6\lambda - 8\lambda - 4k \end{aligned}$$

$$= -\lambda^3 - 5\lambda^2 - 14\lambda - 4k$$

$$\lambda^3 + 5\lambda^2 + 14\lambda + 4k$$

necessary condition ok

$$H = \left[\begin{array}{cc|c} 5 & 4k & 0 \\ 1 & 14 & 0 \\ \hline 0 & 5 & 4k \end{array} \right]$$

$$5 \cdot 14 > 4k$$

$$k < \frac{35}{2} \text{ G.A.S.}$$

$$5) \quad \bar{Y} - \frac{4}{3}\bar{Y} + \frac{1}{2}\bar{Y} = 2$$

$$\frac{6-8+3}{6}\bar{Y} = 2$$

$$\frac{\bar{Y}}{6} = 2$$

$$\bar{Y} = 12$$

$$\lambda^2 - \frac{4}{3}\lambda + \frac{1}{2}$$

$$\frac{4}{3} < 1 + \frac{1}{2} < 2$$

$$\frac{4}{3} < \frac{3}{2} < 2 \quad \checkmark \text{ G.A.S.}$$

$$\lambda_{1/2} = \frac{\frac{4}{3} \pm \sqrt{\frac{16}{9} - 2}}{2} \Rightarrow \Delta < 0$$

$\Rightarrow$ oscillations

