

UNIVERSITÀ CATTOLICA DEL SACRO CUORE

Dynamical Systems in Finance **25th September 2026**

1. A gambler starts from 5€ and aims to reach a total of 15€. If the probability of winning is 0.55, what is the probability that the gambler gets ruined?

What is the probability of ruin if the game is fair ($p = 1/2$)? In this case, what would be the target N needed to have exactly a 75% chance of getting ruined?

2. Discuss the stability of equilibria of the linear non-homogeneous second-order discrete equation

$$x_{h+2} - 4x_{h+1} - 5x_h + 8 = 0$$

and find the explicit solution for $x_0 = 2$, $x_1 = 6$.

3. Discuss the stability of all the equilibria of the nonlinear system of ODEs

$$\begin{cases} \dot{x} = x^2 y - x \\ \dot{y} = \alpha(x - y) \end{cases}$$

for $\alpha \neq 0$. Are there any saddles, nodes or foci?

4. A Phillips' stabilization model with a mixed integral-proportional policy is of the form

$$\begin{cases} \dot{Y} = -2Y + 4G - 8 \\ \dot{G} = -3G - k \int_0^t Y(\tau) d\tau - 2Y \end{cases}$$

for $k > 0$. Write the equilibrium position and discuss its stability.

5. The equation of a Samuelson's accelerator model is given by

$$Y_{h+1} - \frac{4}{3}Y_h + \frac{1}{2}Y_{h-1} = 2.$$

Find the equilibrium position and discuss its stability. Moreover, discuss the existence of oscillations.