

# UNIVERSITÀ CATTOLICA DEL SACRO CUORE

## **Dynamical Systems in Finance** **25th September 2026**

1. A gambler starts from 5€ and aims to reach a total of 15€. If the probability of winning is 0.55, what is the probability that the gambler gets ruined?

What is the probability of ruin if the game is fair ( $p = 1/2$ )? In this case, what would be the target  $N$  needed to have exactly a 75% chance of getting ruined?

2. Discuss the stability of equilibria of the linear non-homogeneous second-order discrete equation

$$x_{h+2} - 4x_{h+1} - 5x_h + 8 = 0$$

and find the explicit solution for  $x_0 = 2$ ,  $x_1 = 6$ .

3. Discuss the stability of all the equilibria of the nonlinear system of ODEs

$$\begin{cases} \dot{x} = x^2 y - x \\ \dot{y} = \alpha(x - y) \end{cases}$$

for  $\alpha \neq 0$ . Are there any saddles, nodes or foci?

4. A Phillips' stabilization model with a mixed integral-proportional policy is of the form

$$\begin{cases} \dot{Y} = -2Y + 4G - 8 \\ \dot{G} = -3G - k \int_0^t Y(\tau) d\tau - 2Y \end{cases}$$

for  $k > 0$ . Write the equilibrium position and discuss its stability.

5. The equation of a Samuelson's accelerator model is given by

$$Y_{h+1} - \frac{4}{3}Y_h + \frac{1}{2}Y_{h-1} = 2.$$

Find the equilibrium position and discuss its stability. Moreover, discuss the existence of oscillations.

# UNIVERSITÀ CATTOLICA DEL SACRO CUORE

## **Dynamical Systems in Finance** **17th July 2026**

1. A gambler starts from 5€ and aims to reach a total of 18€. If the probability of winning is 0.52, what is the probability that the gambler gets ruined?

What happens if the probability of winning lowers to 1/2? In this case, what would be the target  $N$  needed to have exactly a 50% chance of getting ruined?

2. Discuss the stability of equilibria of the linear second-order discrete equation

$$x_{h+2} - x_{h+1} + \alpha x_h = 2$$

for  $\alpha \in \mathbb{R}$ .

In the case  $\alpha = -2$ , find the explicit solution for  $x_0 = 1$ ,  $x_1 = 1$ .

3. The interaction between the price and the supply of a good follows the system of ODEs

$$\begin{cases} \dot{P} = P(4 - P - 2S) \\ \dot{S} = S(3 - 2P - S). \end{cases}$$

Discuss the stability of all the equilibria. Are there any saddles, nodes or foci?

4. A Phillips' stabilization model with a mixed integral-proportional policy is of the form

$$\begin{cases} \dot{Y} = -Y + G + 5 \\ \dot{G} = -3G - \int_0^t Y(\tau) d\tau + kY \end{cases}$$

for  $k > 0$ .

Write the equilibrium position and discuss its stability.

5. Discuss the stability of the equilibria of the d.d.e.

$$x_{h+1} = x_h(1 + \alpha - 2x_h^2), \quad \alpha \in \mathbb{R}.$$

# UNIVERSITÀ CATTOLICA DEL SACRO CUORE

## **Dynamical Systems in Finance** **26th June 2026**

1. A gambler starts from 6€ and aims to reach a total of 20€. If the probability of winning is 0.55, what is the probability that the gambler gets ruined?

What happens if the probability of winning lowers to 1/2? In this case, what would be the target  $N$  needed to have exactly a 25% chance of getting ruined?

2. Discuss the stability of equilibria of the linear second-order discrete equation

$$x_{h+2} + 3x_{h+1} + \alpha x_h = 3$$

for  $\alpha \in \mathbb{R}$ .

In the case  $\alpha = 2$ , find the explicit solution for  $x_0 = 0$ ,  $x_1 = 1$ .

3. The interaction between the price and the supply of a good follows the system of ODEs

$$\begin{cases} \dot{P} = P(3 - P - 2S) \\ \dot{S} = S(2 - P - S). \end{cases}$$

Discuss the stability of all the equilibria. Are there any saddles, nodes or foci?

4. A Phillips' stabilization model with a mixed integral-proportional policy is of the form

$$\begin{cases} \dot{Y} = -3Y + G + 2 \\ \dot{G} = -2G - \int_0^t Y(\tau) d\tau - kY \end{cases}$$

for  $k > 0$ .

Write the equilibrium position and discuss its stability.

5. Discuss the stability of the equilibria of the d.d.e.

$$x_{h+1} = x_h(1 + \alpha - x_h^2), \quad \alpha \in \mathbb{R}.$$

# UNIVERSITÀ CATTOLICA DEL SACRO CUORE

## **Dynamical Systems in Finance** **8th June 2026**

1. A gambler starts from 4€ and aims to reach a total of 15€. If the probability of winning is 0.45, what is the probability that the gambler gets ruined?

What happens if the probability of winning raises to 1/2? In this case, what would be the target  $N$  needed to have exactly a 60% chance of getting ruined?

2. Discuss the stability of equilibria of the linear second-order discrete equation

$$x_{h+2} - \frac{3}{2}x_{h+1} + \alpha x_h = 3$$

for  $\alpha \in \mathbb{R}$ .

In the case  $\alpha = -1$ , find the explicit solution for  $x_0 = 0$ ,  $x_1 = 0$ .

3. The interaction between the price and the supply of a good follows the system of ODEs

$$\begin{cases} \dot{P} = P(2 - P - S) \\ \dot{S} = S(3 - 2P - S). \end{cases}$$

Discuss the stability of all the equilibria. Are there any saddles, nodes or foci?

4. A Phillips' stabilization model with a mixed integral-proportional policy is of the form

$$\begin{cases} \dot{Y} = -2Y + G - 1 \\ \dot{G} = -G - \int_0^t Y(\tau) d\tau - kY \end{cases}$$

for  $k > 0$ .

Write the equilibrium position and discuss its stability.

5. Discuss the stability of the equilibria of the d.d.e.

$$x_{h+1} = x_h(x_h^2 - \alpha), \quad \alpha \in \mathbb{R}.$$

# UNIVERSITÀ CATTOLICA DEL SACRO CUORE

## Dynamical Systems in Finance

February 13, 2026

1. A gambler starts from 10€ and aims to reach a total of 20€. If the probability of winning is 0.4, what is the probability that the gambler gets ruined?

What happens if the probability of winning raises to 1/2? In this case, what would be the target  $N$  needed to have exactly a 60% chance of getting ruined?

2. Discuss the stability of the linear second-order discrete equation

$$x_{h+2} - \frac{1}{2}x_{h+1} + \alpha x_h - 1 = 0$$

for  $\alpha \in \mathbb{R}$ .

In the case  $\alpha = 0$ , find the explicit solution for  $x_0 = 2$ ,  $x_1 = 5$ .

3. The variations of supply and demand of a good follow the system of ODEs

$$\begin{cases} \dot{D} = D(2 - D - S) \\ \dot{S} = S(3 - D - 2S). \end{cases}$$

Discuss the stability of all the equilibria. Are there any saddles, nodes or foci?

4. A Phillips' stabilization model with a mixed integral-proportional policy is of the form

$$\begin{cases} \dot{Y} = -Y + 2G - 6 \\ \dot{G} = -G - k \int_0^t Y(\tau) d\tau - Y \end{cases}$$

for  $k > 0$ .

Write the equilibrium position and discuss its stability.

5. Discuss the stability of the equilibria of the d.d.e.

$$x_{h+1} = \frac{\alpha x_h}{x_h^2 + 1}, \quad \alpha > 0.$$

We recall that the derivative of a rational function  $\frac{f(x)}{g(x)}$  is given by

$$\frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}.$$

# UNIVERSITÀ CATTOLICA DEL SACRO CUORE

## **Dynamical Systems in Finance** **January 16th, 2026**

1. A gambler starts from 4€ and aims to reach a total of 7€. If the probability of winning is  $\frac{2}{3}$ , what is the probability that the gambler gets ruined?

What happens if the probability of winning lowers to  $1/2$ ? In this case, what would be the target  $N$  needed to have exactly a 50% chance of getting ruined?

2. Discuss the stability of the linear second-order discrete equation

$$x_{h+2} - x_{h+1} - \alpha x_h + 1 = 0$$

for  $\alpha \in \mathbb{R}$ .

In the case  $\alpha = 2$ , find the explicit solution for  $x_0 = 1$ ,  $x_1 = 3$ .

3. The variations of supply and demand of a good follow the system of ODEs

$$\begin{cases} \dot{D} = kD + kDS \\ \dot{S} = -kD - S \end{cases}$$

for  $k \neq 0$ .

Discuss the stability of all the equilibria. Are there any saddles, nodes or foci?

4. A Phillips' stabilization model with a mixed integral-proportional policy is of the form

$$\begin{cases} \dot{Y} = -Y + 3G - 3 \\ \dot{G} = -G - k \int_0^t Y(\tau) d\tau - kY \end{cases}$$

for  $k > 0$ .

Write the equilibrium position and discuss its stability.

5. Discuss the stability of the equilibria of the d.d.e.

$$x_{h+1} = \frac{x_h}{x_h^2 + \alpha}, \quad \alpha > 0, \alpha \neq 1.$$

# UNIVERSITÀ CATTOLICA DEL SACRO CUORE

## Dynamical Systems in Finance December 3, 2025

1. A gambler starts from 2€ and aims to reach a total of 5€. If the probability of winning is  $\frac{2}{5}$ , what is the probability that the gambler gets ruined?

What happens if the probability of winning raises to  $1/2$ ? What would be the target  $N$  needed to have exactly a 50% chance of getting ruined?

2. Discuss the stability of the equilibria of the nonlinear discrete equation

$$x_{h+2} - x_{h+1}^2 - \alpha x_{h+1}x_h + \frac{1}{2}x_h = 0$$

for  $\alpha > 0$ .

3. The variations of supply and demand of a good follow the system of ODEs

$$\begin{cases} \dot{D} = D(1 - D - kS) \\ \dot{S} = S(D - k) \end{cases}$$

for  $k \neq 0$ .

Discuss the stability of all the equilibria. Are there any saddles, nodes or foci?

4. A Phillips' stabilization model with a mixed integral-proportional policy is of the form

$$\begin{cases} \dot{Y} = -kY + 3G - 3 \\ \dot{G} = -G - k \int_0^t Y(\tau) d\tau - Y \end{cases}$$

for  $k > 0$ .

Write the equilibrium position and discuss its stability.

5. Discuss the stability of the equilibria of the d.d.e.

$$x_{h+1} = x_h^3 - \alpha x_h^2 + x_h$$

for  $\alpha \in \mathbb{R}$ .

# UNIVERSITÀ CATTOLICA DEL SACRO CUORE

## **Dynamical Systems in Finance** **July 18th, 2025**

1. Discuss the stability of the equilibria of the nonlinear discrete equation

$$x_{h+2} - 2x_{h+1} - x_{h+1}x_h + 2x_h = 0.$$

2. Discuss the stability of all the equilibria of the linear system of ODEs

$$\begin{cases} \dot{x} = \alpha(x - y) \\ \dot{y} = 2x - y \end{cases}$$

for  $\alpha \in \mathbb{R}$ , finding all the saddles, nodes and foci.

3. A Phillips' stabilization model with integral policy is of the form

$$\begin{cases} \dot{Y} = -kY + G - 2 \\ \dot{G} = -3G - \int_0^t Y(\tau) d\tau \end{cases}$$

for  $k > 0$ . Write the equilibrium position and discuss its stability.

4. Discuss the stability of the equilibria of the d.d.s.

$$\begin{cases} x_{h+1} = -x_h^2 - \alpha y_h \\ y_{h+1} = -x_h + y_h \end{cases}$$

for  $\alpha \in \mathbb{R}$ .

5. A gambler starts from 2€ and has a target of winning 5€. If the probability of winning is  $2/3$ , what is the probability of getting ruined?

If the probability of winning drops to  $1/2$ , what is the value  $N$  of the target for which the probability of ruining is exactly 50%?

# UNIVERSITÀ CATTOLICA DEL SACRO CUORE

## **Dynamical Systems in Finance** **Feb 7th, 2025**

1. A nonlinear DDS is of the form

$$\begin{cases} x_{h+1} = x_h(2 - ky_h) \\ y_{h+1} = y_h(x_h - 1) \end{cases}$$

for  $k \in \mathbb{R}$ . Find the equilibria and discuss their stability.

2. Discuss the stability of all the equilibria of the nonlinear system of ODEs

$$\begin{cases} \dot{x} = x^3y^2 + x \\ \dot{y} = -x + \alpha(y - y^2) \end{cases}$$

for  $\alpha \neq 0$ . Are there any saddles, nodes or foci?

3. A Phillips' stabilization model with integral policy is of the form

$$\begin{cases} \dot{Y} = -kY + G - 1 \\ \dot{G} = -2G - \int_0^t Y(\tau)d\tau \end{cases}$$

for  $k > 0$ . Write the equilibrium position and discuss its stability.

4. The equation of a Samuelson's accelerator model is given by

$$Y_{h+1} - \frac{7}{2}Y_h + 3Y_{h-1} = 1.$$

Find the equilibrium position and discuss its stability. Moreover, find the explicit solution such that  $Y_0 = 0$  and  $Y_1 = 2$ .

5. A gambler starts from 1€ and has a target of winning 4€. If the probability of winning is  $2/3$ , what is the probability of getting ruined?

If the probability of winning drops to  $1/2$ , what is the value  $N$  of the target for which the probability of ruining is exactly 80%?

# UNIVERSITÀ CATTOLICA DEL SACRO CUORE

## Dynamical Systems in Finance

Jan 10th, 2025

1. Discuss the stability of the equilibria of the linear non-homogeneous discrete equation

$$x_{h+2} - 2x_{h+1} - 3x_h + 4 = 0$$

and find the explicit solution such that  $x_0 = 0$  and  $x_1 = -2$ .

2. A gambler starts from 2€ and has a target of winning 6€. If the probability of winning is  $2/3$ , what is the probability of getting ruined?

If the probability of winning drops to  $1/2$ , what is the value  $N$  of the target for which the probability of ruining is exactly 50%?

3. Discuss the stability of all the equilibria of the nonlinear system of ODEs

$$\begin{cases} \dot{x} = -x^3 + \alpha y \\ \dot{y} = -x + \alpha y \end{cases}$$

for  $\alpha \neq 0$ . Are there any saddles, nodes or foci?

4. A Phillips' stabilization model with integral policy is of the form

$$\begin{cases} \dot{Y} = -2Y + 3G - 3 \\ \dot{G} = -2G - k \int_0^t Y(\tau) d\tau \end{cases}$$

for  $k > 0$ . Write the equilibrium position and discuss its stability.

5. The equation of a Samuelson's accelerator model is given by

$$Y_{h+1} - \frac{1}{3}(1 + \beta)Y_h + \frac{\beta}{3}Y_{h-1} = 1, \quad \beta > 0.$$

Find the equilibrium position and discuss its stability. Moreover, discuss the existence of oscillations.

# UNIVERSITÀ CATTOLICA DEL SACRO CUORE

## Dynamical Systems in Finance December 9th, 2024

1. Discuss the stability of the equilibria of the nonlinear discrete equation

$$x_{h+2} - 2x_{h+1} - x_{h+1}x_h + 2x_h = 0.$$

2. A gambler starts from 3€ and has a target of winning 5€. If the probability of winning is  $1/3$ , what is the probability of getting ruined?

What happens if the probability of winning raises to  $1/2$ ?

3. Discuss the stability of all the equilibria of the nonlinear system of ODEs

$$\begin{cases} \dot{x} = -x + y \\ \dot{y} = -x^3 + \alpha y \end{cases}$$

for  $\alpha \neq 0$ . Are there any saddles, nodes or foci?

4. A Phillips' stabilization model with mixed integral-proportional policy is of the form

$$\begin{cases} \dot{Y} = -2Y + 3G - 3 \\ \dot{G} = -2G - k \int_0^t Y(\tau) d\tau - \frac{1}{3}Y \end{cases}$$

for  $k > 0$ . Write the equilibrium position and discuss its stability.

5. The dynamics of a price in a Goodwin model is described by the equation

$$\alpha p_{h+2} + (3 - \alpha)p_{h+1} - p_h = 6, \quad 0 < \alpha < 3.$$

Find the equilibrium position and discuss its stability.

In the case  $\alpha = 2$  discuss the stability writing the explicit solutions.

# UNIVERSITÀ CATTOLICA DEL SACRO CUORE

## **Dynamical Systems in Finance** **September 27th, 2024**

1. The dynamics of a price in a Goodwin model is described by the equation

$$2p_{h+2} + \alpha p_{h+1} - \alpha p_h = 2, \quad \alpha > 0.$$

Find the equilibrium position and discuss its stability.

In the case  $\alpha = 1$  discuss the stability and write the explicit solutions.

2. A gambler starts from 4€ and has a target of winning 8€. If the probability of winning is 1/4, what is the probability for the gambler to get ruined?

What happens if the probability of winning raises to 3/4?

3. Discuss the stability of all the equilibria of the linear system of ODEs

$$\begin{cases} \dot{x} = x - y \\ \dot{y} = 2x + \alpha y \end{cases}$$

for  $\alpha \in \mathbb{R}$ . Are there any saddle, node or focus?

4. A Phillips' stabilization model with integral policy is of the form

$$\begin{cases} \dot{Y} = -Y + 3G - 2 \\ \dot{G} = -2G - k \int_0^t Y(\tau) d\tau \end{cases}$$

for  $k > 0$ . Write the equilibrium position and discuss its stability.

5. Discuss the stability of the equilibria of the nonlinear discrete equation

$$x_{h+2} - \frac{3}{\alpha} x_{h+1}^2 + 2x_h = 0$$

for  $\alpha \neq 0$ .

# UNIVERSITÀ CATTOLICA DEL SACRO CUORE

## **Dynamical Systems in Finance** **February 9th, 2024**

1. The dynamics of a price in a Goodwin model is described by the equation

$$3p_{h+2} + (\alpha + 1)p_{h+1} - \alpha p_h = 2, \quad \alpha > 0.$$

Find the equilibrium position and discuss its stability.

In the case  $\alpha = 1$  discuss the stability and write the explicit solutions.

2. A gambler starts from 3€ and has a target of winning 9€. If the probability of winning is  $3/4$ , it is more likely that the gambler get ruined or that he/she reaches the target?

What happens if the probability of winning drops to  $1/2$ ?

3. Discuss the stability of all the equilibria of the linear system of ODEs

$$\begin{cases} \dot{x} = x + y \\ \dot{y} = \alpha x + y \end{cases}$$

for  $\alpha \in \mathbb{R}$ . Are there any saddle, node or focus?

4. A Phillips' stabilization model with integral policy is of the form

$$\begin{cases} \dot{Y} = -\frac{1}{3}Y + 3G - 1 \\ \dot{G} = -kG - k \int_0^t Y(\tau) d\tau \end{cases}$$

for  $k > 0$ . Write the equilibrium position and discuss its stability.

5. Discuss the stability of the equilibria of the nonlinear discrete equation

$$x_{h+1} = \alpha x_h (1 - \alpha x_h^2)$$

for  $\alpha \in \mathbb{R}$ .

# UNIVERSITÀ CATTOLICA DEL SACRO CUORE

## **Dynamical Systems in Finance** **January 19th, 2024**

1. The dynamics of a price in a Goodwin model is described by the equation

$$2p_{h+2} + 2\left(\alpha + \frac{1}{3}\right)p_{h+1} - 2\alpha p_h = 8, \quad \alpha > 0.$$

Find the equilibrium position and discuss its stability.

In the case  $\alpha = 1/3$  discuss the stability and write the explicit solutions.

2. A gambler starts from 3€ and has a target of winning 6€. If the probability of winning is  $3/4$ , it is more likely that the gambler get ruined or that he/she reaches the target?

What happens if the probability of winning drops to  $1/2$ ?

3. Discuss the stability of all the equilibria of the linear system of ODEs

$$\begin{cases} \dot{x} = \alpha x - y \\ \dot{y} = 2\alpha x - 2\alpha y \end{cases}$$

for  $\alpha \in \mathbb{R}$ . Are there any saddle, node or focus?

4. A Phillips' stabilization model with integral policy is of the form

$$\begin{cases} \dot{Y} = -\frac{2}{5}Y + 2G - 2 \\ \dot{G} = -2G - k \int_0^t Y(\tau) d\tau \end{cases}$$

for  $k > 0$ . Write the equilibrium position and discuss its stability.

5. Discuss the stability of the equilibria of the nonlinear discrete equation

$$x_{h+1} = \frac{1 + \alpha x_h^2}{x_h}$$

for  $\alpha \in \mathbb{R}$ .

# UNIVERSITÀ CATTOLICA DEL SACRO CUORE

## **Dynamical Systems in Finance** **December 11th, 2023**

1. The dynamics of a price in a Goodwin model is described by the equation

$$3p_{h+2} + (\alpha + 1)p_{h+1} - \alpha p_h = 4, \quad \alpha > 0.$$

Find the equilibrium position and discuss its stability.

In the case  $\alpha = 1$  discuss the stability and write the explicit solutions.

2. A gambler starts from 1€ and has a target of winning 5€. If the probability of winning is  $2/3$ , it is more likely that the gambler get ruined or that he/she reaches the target?

What happens if the probability of winning drops to  $1/2$ ?

3. Discuss the stability of all the equilibria of the nonlinear system of ODEs

$$\begin{cases} \dot{x} = y^2 \\ \dot{y} = \alpha x + y - z^2 \\ \dot{z} = x - z^2 \end{cases}$$

for  $\alpha \in \mathbb{R}$ .

4. A Phillips' stabilization model with integral policy is of the form

$$\begin{cases} \dot{Y} = -\frac{2}{5}Y + 2G - 2 \\ \dot{G} = -2G - k \int_0^t Y(\tau) d\tau \end{cases}$$

for  $k > 0$ . Write the equilibrium position and discuss its stability.

5. Discuss the stability of the equilibria of the nonlinear discrete equation

$$x_{h+2} - \frac{1}{2}x_{h+1} - x_{h+1}x_h - \frac{1}{4}x_h = 0.$$

# UNIVERSITÀ CATTOLICA DEL SACRO CUORE

## **Dynamical Systems in Finance** **June 30th, 2023**

1. The dynamics of a price in a Goodwin model with regressive expectation is described by the equation

$$2p_{h+2} + p_{h+1} + p_h = 3.$$

Find the equilibrium position and discuss its stability.

2. A gambler wins 1€ with probability  $q = 1/3$  and loses 1€ with probability  $1 - q = 2/3$ . If he/she wants to reach a target of 6€ starting from 3€, what is the probability of getting ruined?

What should be the value of  $q$  in order to lower the probability of getting ruined below 50%?

3. Discuss the stability of the origin in the system of ODEs

$$\begin{cases} \dot{x} = -\alpha x \\ \dot{y} = -\alpha y - z \\ \dot{z} = x - z \end{cases}$$

for  $\alpha \in \mathbb{R}$ . Are there any other equilibrium positions?

4. A Phillips' stabilization model with proportional policy is of the form

$$\begin{cases} \dot{Y} = -3Y + G - 3 \\ \dot{G} = -G - \frac{1}{2}Y \end{cases}$$

Write the equilibrium position and discuss its stability.

# UNIVERSITÀ CATTOLICA DEL SACRO CUORE

## Dynamical Systems in Finance

June 9th, 2023

1. The dynamics of a price in a Goodwin model is described by the equation

$$p_{h+2} + \frac{1}{2}p_{h+1} - \frac{1}{4}p_h = 4.$$

Find the equilibrium position and discuss its stability. Moreover, write all the explicit solutions.

2. What is the probability of a gambler's ruin if the target is to win 6€ and he/she starts from 2€? Assume that the gambler has a 40% chance of winning.

3. Discuss the stability of the origin in the system of ODEs

$$\begin{cases} \dot{x} = -x \\ \dot{y} = \alpha y - z \\ \dot{z} = y - z \end{cases}$$

for  $\alpha \in \mathbb{R}$ . Are there any other equilibrium positions?

4. A Phillips' stabilization model with integral policy is of the form

$$\begin{cases} \dot{Y} = -\frac{1}{3}Y + G - 2 \\ \dot{G} = -G - \frac{1}{5} \int_0^t Y(\tau) d\tau. \end{cases}$$

Write the equilibrium position and discuss its stability.

5. Discuss the stability of the equilibria of the d.d.e.

$$x_{h+1} = \alpha x_h^3 + x_h$$

for  $\alpha \in \mathbb{R}$ .

# UNIVERSITÀ CATTOLICA DEL SACRO CUORE

## **Dynamical Systems in Finance** **January 13th, 2023**

1. The dynamics of a price in a Goodwin model is described by the equation

$$p_{h+2} + \frac{2}{3}p_{h+1} - \frac{1}{3}p_h = 4.$$

Find the equilibrium position and discuss its stability. Moreover, write all the explicit solutions.

2. What is the probability of a gambler's ruin if the target is to win 4€ and he/she starts from 2€? Assume that the gambler has a 60% chance of winning.

3. Discuss the stability of the origin in the system of ODEs

$$\begin{cases} \dot{x} = \alpha x \\ \dot{y} = -2y - 3z \\ \dot{z} = 2y + z \end{cases}$$

for  $\alpha \in \mathbb{R}$ . Are there any other equilibrium positions?

4. A Phillips' stabilization model with integral policy is of the form

$$\begin{cases} \dot{Y} = -\frac{1}{2}Y + G - 3 \\ \dot{G} = -2G - 2 \int_0^t Y(\tau) d\tau. \end{cases}$$

Write the equilibrium position and discuss its stability.

5. Discuss the stability of the equilibria of the d.d.e.

$$x_{h+1} = x_h^2 + \alpha x_h$$

for  $\alpha \in \mathbb{R}$ .

# UNIVERSITÀ CATTOLICA DEL SACRO CUORE

## **Dynamical Systems in Finance** **December 19th, 2022**

1. The dynamics of a price in a Goodwin model is described by the equation

$$p_{h+2} + \frac{4}{3}p_{h+1} - \frac{1}{4}p_h = 10.$$

Find the equilibrium position and discuss its stability. Moreover, write all the explicit solutions.

2. What is the probability of a gambler's ruin if the target is to win 150€ and he/she starts from 20€? Assume that the game is fair.

3. Discuss the stability of the origin in the system of ODEs

$$\begin{cases} \dot{x} = y \\ \dot{y} = -2x - 3y \\ \dot{z} = -\alpha z \end{cases}$$

for  $\alpha \in \mathbb{R}$ . Are there any other equilibrium positions?

4. A Phillips' stabilization model with mixed policy is of the form

$$\begin{cases} \dot{Y} = -3Y + G - 2 \\ \dot{G} = -G - 2Y - 2\dot{Y} \end{cases}$$

Write the equilibrium position and discuss its stability.

5. Discuss the stability of the equilibria of the d.d.s.

$$\begin{cases} x_{h+1} = x_h - y_h \\ y_{h+1} = \alpha x_h - y_h^2 \end{cases}$$

for  $\alpha \in \mathbb{R}$ .

## Dynamical Systems in Finance - Written test - February 4, 2022

1. Find the equilibrium of the linear discrete dynamical system

$$\begin{cases} x_{h+1} = x_h + 2y_h - 1 \\ y_{h+1} = 3x_h + 7y_h - 3 \end{cases}$$

and discuss its stability.

2. The dynamics of price variations  $P_1, P_2 \in \mathbb{R}$  of two goods is described by the nonlinear system

$$\begin{cases} \dot{P}_1 = P_1(kP_2 - 2) \\ \dot{P}_2 = -P_2(2P_1 + 1). \end{cases}$$

where  $k \neq 0$ . Find the equilibrium positions and discuss their stability.

3. A gambler plays a game where he/she can win 1€ with probability 1/3. What is the probability of the gambler's ruin if the target is to reach 5€ and he/she starts from 2€?

4. A Phillips' stabilization model with integral policy is of the form

$$\begin{cases} \dot{Y} = -\frac{1}{2}Y + G - 1 \\ \dot{G} = -2G - \int_0^t Y(\tau) d\tau. \end{cases}$$

Write the equilibrium position and discuss its stability.

5. Consider the discrete map

$$x_{h+1} = x_h + 1 - \sqrt{x_h}$$

defined for  $x_0 > 0$ . Find the equilibria (if any) and discuss their stability.

## Dynamical Systems in Finance - Written test - January 14, 2022

1. The dynamics of a price in a Cobweb model is described by the equations

$$D_h = -\frac{1}{2}p_h + 1, \quad S_{h+1} = p_h + \frac{1}{2}$$

where  $D$  is the demand and  $S$  the supply.

Find the equilibrium position and discuss its stability. Moreover find the price at time  $h = 4$  supposing that  $p_0 = 1$ .

2. The dynamics of price variations  $P_1, P_2 \in \mathbb{R}$  of two goods is described by the nonlinear system

$$\begin{cases} \dot{P}_1 = P_1(1 - P_2) \\ \dot{P}_2 = P_2(P_1 + k). \end{cases}$$

where  $k > 0$ . Find the equilibrium positions and discuss their stability.

3. A gambler plays a game where he/she can win 1€ with probability  $2/3$ . What is the probability of the gambler's ruin if the target is to reach 5€ and he/she starts from 2€?

4. A Phillips' stabilization model with integral policy is of the form

$$\begin{cases} \dot{Y} = -\frac{2}{3}Y + G - 2 \\ \dot{G} = -G - \int_0^t Y(\tau)d\tau. \end{cases}$$

Write the equilibrium position and discuss its stability.

5. Consider the discrete map

$$x_{h+1} = x_h - \frac{8}{x_h}.$$

Find the equilibria (if any) and discuss their stability. Moreover, find the 2-cycle.

What happens if the number 8 is replaced by a parameter  $d > 0$ ?