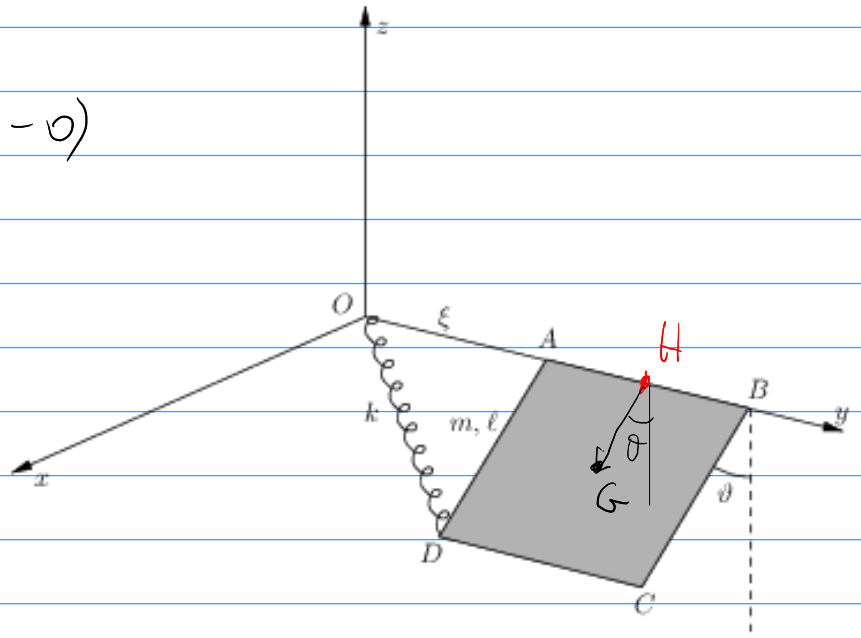
MA 17/1/25

1) $(G-O) = (G-H) + (H-O)$



$$(G-H) = \left(\frac{\ell}{2}\sin\vartheta,\ 0,\ -\frac{\ell}{2}\cos\vartheta\right)$$

$$(H-O) = \left(0,\ \xi+\frac{\ell}{2},\ 0\right)$$

$$(G-O) = \left(\frac{\ell}{2}\sin\vartheta,\ \xi+\frac{\ell}{2},\ -\frac{\ell}{2}\cos\vartheta\right)$$

$$(D-O) = (D-A)+(A-O) = \left(\ell\sin\vartheta,\ \xi,\ -\ell\cos\vartheta\right)$$

$$U = mg\frac{\ell}{2}\cos\vartheta - \frac{1}{2}k\left(\ell^2+\xi^2\right)$$

$$\begin{cases} \dfrac{\partial U}{\partial \xi} = -k\xi = 0 & \xi = 0 \\ \dfrac{\partial U}{\partial \vartheta} = -mg\dfrac{\ell}{2}\sin\vartheta = 0 & \vartheta = 0,\ \pi \end{cases}$$

2) $$H = \begin{bmatrix} -k & 0 \\ 0 & -mg\dfrac{\ell}{2}\cos\vartheta \end{bmatrix}$$

$$H(0,0) = \begin{bmatrix} -k & 0 \\ 0 & -mg\frac{l}{2} \end{bmatrix}$$ STABILE

$$H(0,\pi) = \begin{bmatrix} -k & 0 \\ 0 & mg\frac{l}{2} \end{bmatrix}$$ INSTABILE

3) $K = \frac{1}{2} m v_H^2 + \frac{1}{2} \omega \cdot J_H \omega \qquad (v_H \parallel \omega)$

$v_H^2 = \dot{\xi}^2$

$\omega = -\dot{\theta} e_y \qquad \omega \cdot J_H \omega = I_H \omega^2$

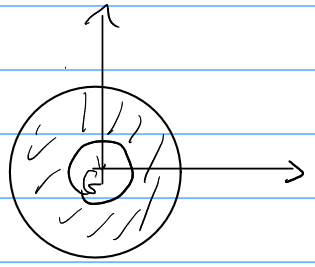
$I_H = \frac{1}{3} m l^2$

$\Rightarrow K = \frac{1}{2} m \dot{\xi}^2 + \frac{1}{2} \cdot \frac{1}{3} m l^2 \dot{\theta}^2$

$L = K + U$

$$K = \begin{bmatrix} m & 0 \\ 0 & \frac{1}{3} m l^2 \end{bmatrix}$$

II



Disco grande: $4m, 2R$

Buco: m, R

$$J_G^D = \begin{bmatrix} \frac{1}{4} 4m 4R^2 & 0 & 0 \\ 0 & 4mR^2 & 0 \\ 0 & 0 & 8mR^2 \end{bmatrix}$$

$$J_G^B = \begin{bmatrix} \frac{mR^2}{4} & 0 & 0 \\ 0 & \frac{mR^2}{4} & 0 \\ 0 & 0 & \frac{mR^2}{2} \end{bmatrix}$$

$$J_G = \begin{bmatrix} \frac{15}{4}mR^2 & 0 & 0 \\ 0 & \frac{15}{4}mR^2 & 0 \\ 0 & 0 & \frac{15}{2}mR^2 \end{bmatrix}$$

Steiner: $\vec{d} = (G-O) = (2R, 2R, 0)$

$$|d|^2 = 8R^2 \qquad d \otimes d = \begin{bmatrix} 4R^2 & 4R^2 & 0 \\ 4R^2 & 4R^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$3m\left(|d|^2 I - d \otimes d\right) = \begin{bmatrix} 12mR^2 & -12mR^2 & 0 \\ -12mR^2 & 12mR^2 & 0 \\ 0 & 0 & 24mR^2 \end{bmatrix}$$

$$J_O = J_G + 3m\left(|d|^2 I - d \otimes d\right) = \begin{bmatrix} \frac{63}{4}mR^2 & -12mR^2 & 0 \\ -12mR^2 & \frac{63}{4}mR^2 & 0 \\ 0 & 0 & \frac{63}{2}mR^2 \end{bmatrix}$$

□