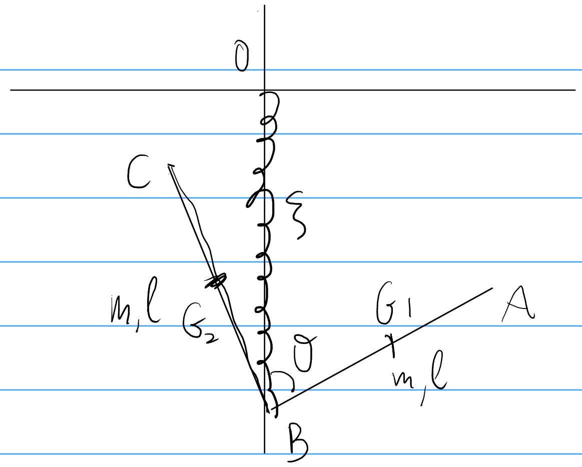
TEMA 07/02/25



$$(B-O)=(0,-\xi)$$

$$(G_1-B)=\left(\frac{l}{2}\sin\vartheta,\ \frac{l}{2}\cos\vartheta\right)$$

$$(G_2-B)=\left(-\frac{l}{2}\cos\vartheta,\ \frac{l}{2}\sin\vartheta\right)$$

$$\lambda=\frac{mg}{kl}$$

$$(G_1-O)=\left(\frac{l}{2}\sin\vartheta,\ -\xi+\frac{l}{2}\cos\vartheta\right)$$

$$(G_2-O)=\left(-\frac{l}{2}\cos\vartheta,\ -\xi+\frac{l}{2}\sin\vartheta\right)$$

$$|B-O|^2=\xi^2$$

$$U=-mg\left(-\xi+\frac{l}{2}\cos\vartheta-\xi+\frac{l}{2}\sin\vartheta\right)-\frac{k}{2}\xi^2$$

$$=2mg\xi-\frac{1}{2}mgl(\sin\vartheta+\cos\vartheta)-\frac{1}{2}k\xi^2$$

$$\frac{\partial U}{\partial \xi}=2mg-k\xi \qquad =0$$

$$\frac{\partial U}{\partial \vartheta}=-\frac{1}{2}mgl(\cos\vartheta-\sin\vartheta) \qquad =0$$

$$\xi = \frac{2mg}{k}$$

$$\sin\vartheta = \cos\vartheta \quad \begin{cases} \vartheta = \frac{\pi}{4} \\ \vartheta = \frac{3}{4}\pi \end{cases}$$

Stabilità:

$$\mathcal{H} = \begin{bmatrix} -k & 0 \\ 0 & \frac{1}{2}mgl(\sin\vartheta + \cos\vartheta) \end{bmatrix}$$

$$\mathcal{H}\left(\frac{2mg}{k}, \frac{\pi}{4}\right) = \begin{bmatrix} -k & 0 \\ 0 & \frac{\sqrt{2}}{2}mgl \end{bmatrix} \quad \text{INST}$$

$$\mathcal{H}\left(\frac{2mg}{k}, \frac{3}{4}\pi\right) = \begin{bmatrix} -k & 0 \\ 0 & -\frac{\sqrt{2}}{2}mgl \end{bmatrix} \quad \text{STAB.}$$

$$K = \frac{1}{2}m v_{G_1}^2 + \frac{1}{2}m v_{G_2}^2 + \frac{1}{2}J_{33}^1\dot\vartheta^2 + \frac{1}{2}J_{33}^2\dot\vartheta^2$$

$$v_{G_1} = \left(\frac{l}{2}\dot\vartheta\cos\vartheta, -\dot\xi - \frac{l}{2}\dot\vartheta\sin\vartheta\right) \quad |v_{G_1}|^2 = \frac{l^2}{4}\dot\vartheta^2 + \dot\xi^2 + l\dot\xi\dot\vartheta\sin\vartheta$$

$$v_{G_2} = \left(\frac{l}{2}\dot\vartheta\sin\vartheta, -\dot\xi + \frac{l}{2}\dot\vartheta\cos\vartheta\right) \quad |v_{G_2}|^2 = \frac{l^2}{4}\dot\vartheta^2 + \dot\xi^2 - l\dot\xi\dot\vartheta\cos\vartheta$$

$$J_{33}^1 = J_{33}^2 = \frac{1}{12}ml^2$$

$$K = \frac{1}{2} m\left(\frac{\ell^2}{2}\dot{\vartheta}^2 + 2\dot{\xi}^2 + \ell\dot{\xi}\dot{\vartheta}(\sin\vartheta - \cos\vartheta)\right) +$$

$$+ \frac{1}{2}\,\frac{1}{6} m\ell^2\dot{\vartheta}^2$$

$$\mathbb{K} = \begin{bmatrix} 2m & \frac{\ell}{2}(\sin\vartheta - \cos\vartheta) \\ \frac{\ell}{2}(\sin\vartheta - \cos\vartheta) & \frac{2}{3}m\ell^2 \end{bmatrix}$$

$$P\left(\frac{2mg}{k}, \frac{3}{4}\pi\right) \qquad \mathcal{H} = \begin{bmatrix} -k & 0 \\ 0 & -\frac{\sqrt{2}}{2}mg\ell \end{bmatrix}$$

$$\tilde{\mathcal{L}} = \frac{1}{2}\dot{q}\,\mathbb{K}\,\dot{q} + \frac{1}{2}(q-\bar{q})\,\mathcal{H}\,(q-\bar{q})$$

$$= \frac{1}{2}\begin{bmatrix}\dot{\xi} & \dot{\vartheta}\end{bmatrix}\begin{bmatrix} 2m & \frac{\sqrt{2}\ell}{4} \\ \frac{\sqrt{2}\ell}{4} & \frac{2}{3}m\ell^2 \end{bmatrix}\begin{bmatrix}\dot{\xi} \\ \dot{\vartheta}\end{bmatrix} + \frac{1}{2}\begin{bmatrix}\xi - \frac{2mg}{k} & \vartheta - \frac{3}{4}\pi\end{bmatrix}\mathcal{H}\begin{bmatrix}\xi - \frac{2mg}{k} \\ \vartheta - \frac{3}{4}\pi\end{bmatrix}$$

$\square$

$\underline{\mathrm{II}}$

$$\begin{cases} Q = kq^\alpha e^{-p} \\ P = kq^\alpha e^{p} \end{cases} \qquad \begin{array}{l} k>0 \\ \alpha\in\mathbb{R} \end{array}$$

$$1 = [Q,P] = k\alpha q^{\alpha-1}\cancel{e^{-p}}\,kq^\alpha\cancel{e^{p}} + kq^\alpha\cancel{e^{-p}}\,k\alpha q^{\alpha-1}\cancel{e^{p}}$$

$$= 2k^2\alpha\, q^{2\alpha-1} \qquad = 1$$

$$\Rightarrow \quad 2\alpha = 1 \qquad \alpha = \frac{1}{2} \qquad \frac{\cancel{2}k^2}{\cancel{2}} = 1 \qquad k = 1$$

$F_2(q, P)$

$$\begin{cases} p = \dfrac{\partial F_2}{\partial q} \\[2ex] Q = \dfrac{\partial F_2}{\partial P} = \dfrac{q}{2P} \end{cases}$$

$$e^p = \frac{P}{kq^\alpha} \qquad Q = \frac{kq^\alpha}{p} \quad kq^\alpha = \frac{q}{p} = \frac{q}{P}$$

$$e^p = \frac{P}{q^{1/2}} \qquad p = \ln\left(\frac{P}{q^{1/2}}\right)$$

$$F_2 = q \ln P + g(q)$$

$$\frac{\partial F_2}{\partial q} = \cancel{\ln P} + g'(q) = \cancel{\ln P} - \ln\left(q^{1/2}\right)$$

$$g'(q) = -\frac{1}{2}\ln q$$

$$g(q) = -\frac{1}{2}\left(q \ln q - q\right) + c = -\frac{1}{2} q \ln q + \frac{q}{2} + c$$

$$\Rightarrow F_2(q, P) = q \ln P - \frac{1}{2} q \ln q + \frac{q}{2} + c$$

$$= q\left(\ln \frac{P}{\sqrt{q}} + \frac{1}{2}\right) + c \qquad \square$$