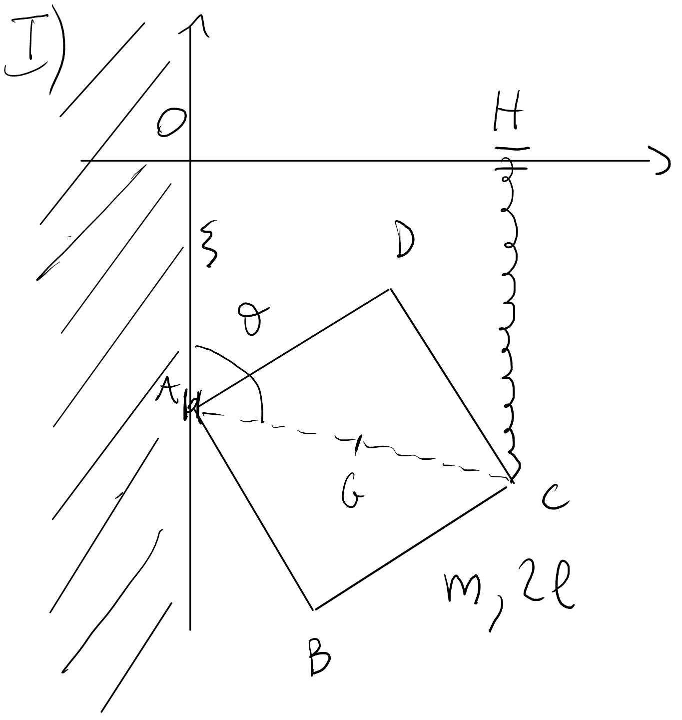
MA 5/9/25

I)



$\theta \in \left[\frac{\pi}{4}, \frac{3}{4}\pi\right]$

$(G-O) = (G-A) + (A-O)$

$= \left(l\sqrt{2}\sin\theta,\ l\sqrt{2}\cos\theta + \xi\right)$

$(C-O) = (C-A) + (A-O)$

$= \left(2l\sqrt{2}\sin\theta,\ 2l\sqrt{2}\cos\theta + \xi\right)$

$|C-H|^2 = 8l^2\cos^2\theta + \xi^2 + 4l\sqrt{2}\,\xi\cos\theta$

$U = -mgl\sqrt{2}\cos\theta - mg\xi - 4kl^2\cos^2\theta - \frac{1}{2}k\xi^2 - 2kl\sqrt{2}\,\xi\cos\theta$

$$\begin{cases} \dfrac{\partial U}{\partial \xi} = -mg - k\xi - 2\sqrt{2}\,kl\cos\theta = 0 \\ \dfrac{\partial U}{\partial \theta} = mg\cancel{l}\sqrt{2}\sin\theta + 8kl^{\cancel{2}}\sin\theta\cos\theta + 2\sqrt{2}\,k\cancel{l}\xi\sin\theta = 0 \end{cases}$$

$\sin\theta\left(mg\sqrt{2} + 8kl\cos\theta + 2\sqrt{2}\,k\xi\right) = 0$

$\theta = 0$ N.A. $\qquad \theta = \pi$ N.A.

$$\begin{cases} mg + k\xi + 2\sqrt{2}\,kl\cos\theta = 0 \\ mg + 2k\xi + \dfrac{8}{\sqrt{2}}\,kl\cos\theta = 0 \end{cases}$$

$4\sqrt{2}$

$\nexists$ pos. eq. ordinarie !

confine

$\theta = \frac{\pi}{4}$
$$\begin{cases} -mg - k\xi - \cancel{2}\sqrt{2}kl \frac{\sqrt{2}}{\cancel{2}} = 0 \\ mg\sqrt{2}\frac{\sqrt{2}}{2} + 8hl\frac{1}{2} + \cancel{2}\sqrt{2}k\xi\frac{\sqrt{2}}{\cancel{2}} \leq 0 \end{cases}$$

$$-mg - k\xi - 2kl = 0 \qquad \xi = -\frac{mg}{k} - 2l$$

$$mg + 4hl + 2k\left(-\frac{mg}{k} - 2l\right) \leq 0$$

$$mg + \cancel{4kl} - 2mg - \cancel{4kl} \leq 0 \qquad \checkmark$$

$$\left(-\frac{mg}{k} - 2l, \frac{\pi}{4}\right) \quad \text{eq. confine}$$

$\theta = \frac{3}{4}\pi$
$$\begin{cases} -mg - k\xi + \cancel{2}\sqrt{2}hl\frac{\sqrt{2}}{\cancel{2}} = 0 \\ mg\sqrt{2}\frac{\sqrt{2}}{2} - 8kl\frac{1}{2} + \cancel{2}\sqrt{2}k\xi\frac{\sqrt{2}}{\cancel{2}} \geq 0 \end{cases}$$

$$-mg - k\xi + 2hl = 0 \qquad \xi = -\frac{mg}{k} + 2l$$

$$mg - 4kl + 2k\left(-\frac{mg}{k} + 2l\right) \geq 0$$

$$mg - \cancel{4kl} - 2mg + \cancel{4kl} \geq 0 \qquad \text{NO}$$

$$K = \frac{1}{2}m\,v_G^2 + \frac{1}{2}J_G^{33}\dot{\theta}^2$$

$$J_G^{33} = \frac{1}{6}m\,4l^2 = \frac{2}{3}ml^2$$

$$\vec{V}_G = \left(\ell\sqrt{2}\,\dot\theta\cos\theta,\ \dot\xi - \ell\sqrt{2}\,\dot\theta\sin\theta\right)$$

$$|V_G|^2 = 2\ell^2\dot\theta^2 + \dot\xi^2 - 2\sqrt{2}\,\ell\,\dot\xi\,\dot\theta\sin\theta$$

$$K = \frac{1}{2}\left(m\dot\xi^2 + 2m\ell^2\dot\theta^2 - 2\sqrt{2}\,m\ell\,\dot\xi\,\dot\theta\sin\theta + \frac{2}{3}m\ell^2\dot\theta^2\right)$$

$$\mathbb{K} = \begin{bmatrix} m & -\sqrt{2}\,m\ell\sin\theta \\ -\sqrt{2}\,m\ell\sin\theta & \frac{8}{3}m\ell^2 \end{bmatrix}$$

II

$$\begin{cases} Q = \alpha p q \\ P = \ln\left(\frac{p}{q^2}\right) \end{cases}$$

$$[Q, P] = \alpha\cancel{p}\,\frac{\cancel{q^2}}{\cancel{p}}\,\frac{1}{\cancel{q^2}} - \alpha q\,\frac{\cancel{q^2}}{\cancel{p}}\,\cancel{p}\left(-\frac{2\cancel{q}}{\cancel{q^3}}\right)$$

$$= 3\alpha = 1 \qquad \alpha = \frac{1}{3}$$

$$F(q, P) \qquad \begin{cases} p = \frac{\partial F}{\partial q} \\ Q = \frac{\partial F}{\partial P} \end{cases}$$

$$Q = \frac{1}{3} pq \qquad Q = \frac{1}{3} q^3 e^P$$

$$e^P = \frac{p}{q^2} \qquad p = q^2 e^P = \frac{\partial F}{\partial q}$$

$$F = \frac{1}{3} q^3 e^P + g(P)$$

$$Q = \frac{\partial F}{\partial P} = \cancel{\frac{1}{3} q^3 e^P} + g'(P) = \cancel{\frac{1}{3} q^3 e^P}$$

$$F_2(q, P) = \frac{1}{3} q^3 e^P + \text{cost.}$$

$\square$