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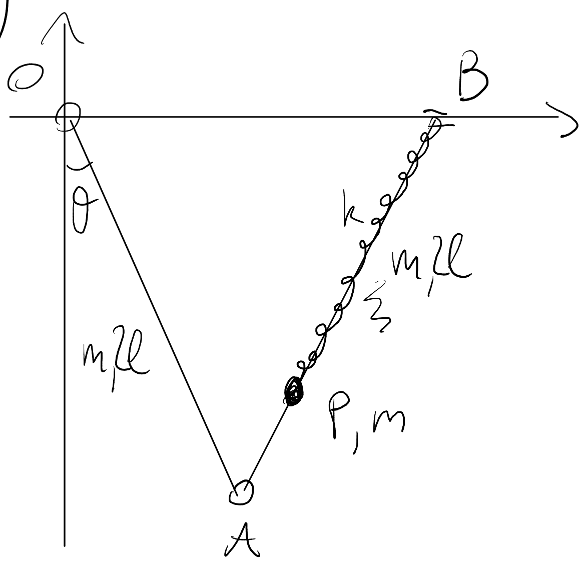
I)




$\xi \in [0, 2\ell]$ $\qquad \lambda = \frac{mg}{k\ell}$

$(G_1 - O) = (\ell \sin\theta, -\ell\cos\theta)$

$$\begin{aligned}(G_2 - O) &= (G_2 - A) + (A - O) \\ &= (\ell\sin\theta, \ell\cos\theta) + (2\ell\sin\theta, -2\ell\cos\theta) \\ &= (3\ell\sin\theta, -\ell\cos\theta)\end{aligned}$$

$$\begin{aligned}(P - O) = (P - A) + (A - O) &= \big((2\ell - \xi)\sin\theta, (2\ell-\xi)\cos\theta\big) \\ &\quad + (2\ell\sin\theta, -2\ell\cos\theta) \\ &= \big((4\ell - \xi)\sin\theta, -\xi\cos\theta\big)\end{aligned}$$

$|P - B|^2 = \xi^2$

$U = 2mg\ell\cos\theta + mg\xi\cos\theta - \frac{1}{2}k\xi^2$

$$\begin{cases}\dfrac{\partial U}{\partial \xi} = mg\cos\theta - k\xi \\[2mm] \dfrac{\partial U}{\partial \theta} = -2mg\ell\sin\theta - mg\xi\sin\theta\end{cases}$$

$\xi = \frac{mg}{k}\cos\theta = \lambda\ell\cos\theta$

$\sin\theta = 0 \qquad \theta = 0 \qquad \xi = \lambda\ell \quad$ se $\quad \lambda < 2$

$\theta = \pi \qquad \xi = -\lambda\ell \quad$ N. A.

$-2\cancel{mg}l - \cancel{mg}\,\xi = 0 \qquad \xi = -2l \quad$ N.A.

$P(\lambda l, 0) \quad$ per $\quad \lambda < 2$

$$H = \begin{bmatrix} -k & -mg\sin\vartheta \\ -mg\sin\vartheta & -2mgl\cos\vartheta - mg\,\xi\cos\vartheta \end{bmatrix}$$

$$H(P) = \begin{bmatrix} -k & 0 \\ 0 & -2mgl - \lambda mgl \end{bmatrix} \qquad \text{STABILE}$$

Confine:

$\xi = 0 \qquad \begin{cases} mg\cos\vartheta \leq 0 \\ -2mgl\sin\vartheta = 0 \end{cases}$

$\vartheta = 0 \quad$ NO

$\vartheta = \pi \quad$ ok $\qquad (0, \pi)$ pos. eq. confine

$\xi = 2l \qquad \begin{cases} mg\cos\vartheta - 2kl \geq 0 \\ -2mgl\sin\vartheta - 2mgl\sin\vartheta = 0 \end{cases}$

$\vartheta = 0 \qquad mg - 2kl \geq 0 \qquad \lambda \geq 2$

$\vartheta = \pi \qquad -mg - 2kl \geq 0 \quad$ N.A.

$(2\ell, 0)$ eq. conf.ne per $\lambda \geq 2$

$$\vec{v}_P = \left(-\dot{\xi}\sin\vartheta + (4\ell-\xi)\dot{\vartheta}\cos\vartheta,\ -\dot{\xi}\cos\vartheta + \xi\dot{\vartheta}\sin\vartheta\right)$$

$$|\vec{v}_P|^2 = \dot{\xi}^2 + (4\ell-\xi)^2\dot{\vartheta}^2\cos^2\vartheta - 2(4\ell-\xi)\dot{\xi}\dot{\vartheta}\sin\vartheta\cos\vartheta$$
$$-2\xi\dot{\xi}\dot{\vartheta}\sin\vartheta\cos\vartheta + \xi^2\dot{\vartheta}^2\sin^2\vartheta$$
$$= \dot{\xi}^2 + 16\ell^2\dot{\vartheta}^2\cos^2\vartheta - 8\ell\xi\dot{\vartheta}^2\cos^2\vartheta + \xi^2\dot{\vartheta}^2\cos^2\vartheta$$
$$-8\ell\xi\dot{\xi}\dot{\vartheta}\sin\vartheta\cos\vartheta + \cancel{2\xi\dot{\xi}\dot{\vartheta}\sin\vartheta\cos\vartheta}$$
$$-\cancel{2\xi\dot{\xi}\dot{\vartheta}}\sin\vartheta\cos\vartheta + \xi^2\dot{\vartheta}^2\sin^2\vartheta$$
$$= \dot{\xi}^2 + \xi^2\dot{\vartheta}^2 + (16\ell^2 - 8\ell\xi)\dot{\vartheta}^2\cos^2\vartheta$$
$$-8\ell\xi\dot{\xi}\dot{\vartheta}\sin\vartheta\cos\vartheta$$

$$\vec{v}_{G_2} = (3\ell\dot{\vartheta}\cos\vartheta,\ -\ell\dot{\vartheta}\sin\vartheta)$$

$$|\vec{v}_{G_2}|^2 = 9\ell^2\dot{\vartheta}^2\cos^2\vartheta + \ell^2\dot{\vartheta}^2\sin^2\vartheta$$
$$= 8\ell^2\dot{\vartheta}^2\cos^2\vartheta + \ell^2\dot{\vartheta}^2$$
$$= \ell^2\dot{\vartheta}^2(1 + 8\cos^2\vartheta)$$

$$K = \frac{1}{2}J_A\omega^2 + \frac{m}{2}|v_{G_2}|^2 + \frac{1}{2}J_{G_2}\omega^2 + \frac{1}{2}m|v_P|^2$$

$$J_A = \frac{4}{3}m\ell^2 \qquad J_{G_2} = \frac{1}{3}m\ell^2$$

$$K = \frac{1}{2}\left(\frac{4}{3}m\ell^2\dot{\vartheta}^2 + m\ell^2\dot{\vartheta}^2(1+8\cos^2\vartheta) + \frac{1}{3}m\ell^2\dot{\vartheta}^2\right.$$

$$+m\dot{\xi}^2 + m\xi^2\dot{\vartheta}^2 + 8ml(2l-\xi)\dot{\vartheta}^2\cos^2\vartheta$$
$$-8ml\xi\dot{\xi}\dot{\vartheta}\sin\vartheta\cos\vartheta\Big)$$

$$=\frac{1}{2}\Big(m\dot{\xi}^2 + m\left(\frac{8}{3}+8\cos^2\vartheta\right)l^2\dot{\vartheta}^2 + m\xi^2\dot{\vartheta}^2$$
$$+8ml(2l-\xi)\dot{\vartheta}^2\cos^2\vartheta$$
$$-8ml\xi\dot{\xi}\dot{\vartheta}\sin\vartheta\cos\vartheta\Big)$$

$$\mathbb{K}=\begin{bmatrix} m & -4ml\xi\sin\vartheta\cos\vartheta \\ -4ml\xi\sin\vartheta\cos\vartheta & m\left(\left(\frac{8}{3}+8\cos^2\vartheta\right)l^2+m\xi^2+8ml(2l-\xi)\cos^2\vartheta\right) \end{bmatrix}$$

II)

$$J_G^{\varphi}=\begin{bmatrix} \frac{ml^2}{12} & & \\ & \frac{ml^2}{12} & \\ & & \frac{ml^2}{6} \end{bmatrix} \qquad \vec{d}=\left(\frac{l}{\sqrt{2}},\frac{l}{\sqrt{2}},0\right)$$

$$|d|^2=l^2 \qquad d\otimes d=\begin{bmatrix} \frac{l^2}{2} & \frac{l^2}{2} & 0 \\ \frac{l^2}{2} & \frac{l^2}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad d^2 I - d\otimes d=$$
$$=\begin{bmatrix} \frac{l^2}{2} & -\frac{l^2}{2} & 0 \\ -\frac{l^2}{2} & \frac{l^2}{2} & 0 \\ 0 & 0 & l^2 \end{bmatrix}$$

$$J_O^{Q} = J_G^{Q} + m\left(d^2 I - d\otimes d\right) =$$

$$= \begin{bmatrix} \frac{7}{12} m\ell^2 & -\frac{m\ell^2}{2} & 0 \\ -\frac{m\ell^2}{2} & \frac{7}{12} m\ell^2 & 0 \\ 0 & 0 & \frac{7}{6} m\ell^2 \end{bmatrix}$$

$$J_G^{a_1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{m\ell^2}{12} & 0 \\ 0 & 0 & \frac{m\ell^2}{12} \end{bmatrix} \qquad d = \left(-\frac{\ell}{2}, \frac{\ell}{\sqrt{2}}, 0\right)$$

$$d^2 = \frac{3}{4}\ell^2 \qquad d\otimes d = \begin{bmatrix} \frac{\ell^2}{4} & -\frac{\ell^2}{2\sqrt{2}} & 0 \\ -\frac{\ell^2}{2\sqrt{2}} & \frac{\ell^2}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$d^2 I - d\otimes d = \begin{bmatrix} \frac{\ell^2}{2} & +\frac{\ell^2}{2\sqrt{2}} & 0 \\ +\frac{\ell^2}{2\sqrt{2}} & \frac{\ell^2}{4} & 0 \\ 0 & 0 & \frac{3}{4}\ell^2 \end{bmatrix}$$

$$J_O^{a_1} = \begin{bmatrix} m\frac{\ell^2}{2} & +\frac{m\ell^2}{2\sqrt{2}} & 0 \\ +\frac{m\ell^2}{2\sqrt{2}} & \frac{m\ell^2}{3} & 0 \\ 0 & 0 & \frac{5}{6} m\ell^2 \end{bmatrix}$$

$$\frac{1}{2\sqrt{2}} - \frac{1}{2} = \frac{1}{2}\left(\frac{\sqrt{2}}{2} - 1\right) = \frac{\sqrt{2}-2}{4}$$

$$J_O^{a_2} = \begin{bmatrix} \frac{m\ell^2}{3} & \frac{m\ell^2}{2\sqrt{2}} & 0 \\ \frac{m\ell^2}{2\sqrt{2}} & \frac{m\ell^2}{2} & 0 \\ 0 & 0 & \frac{5}{6} m\ell^2 \end{bmatrix}$$

$$J_O^{TOT} = \begin{bmatrix} \frac{17}{12} m l^2 & \frac{\sqrt{2}-2}{4} m l^2 & 0 \\ \frac{\sqrt{2}-2}{4} m l^2 & \frac{17}{12} m l^2 & 0 \\ 0 & 0 & \frac{17}{6} m l^2 \end{bmatrix}$$

$\square$