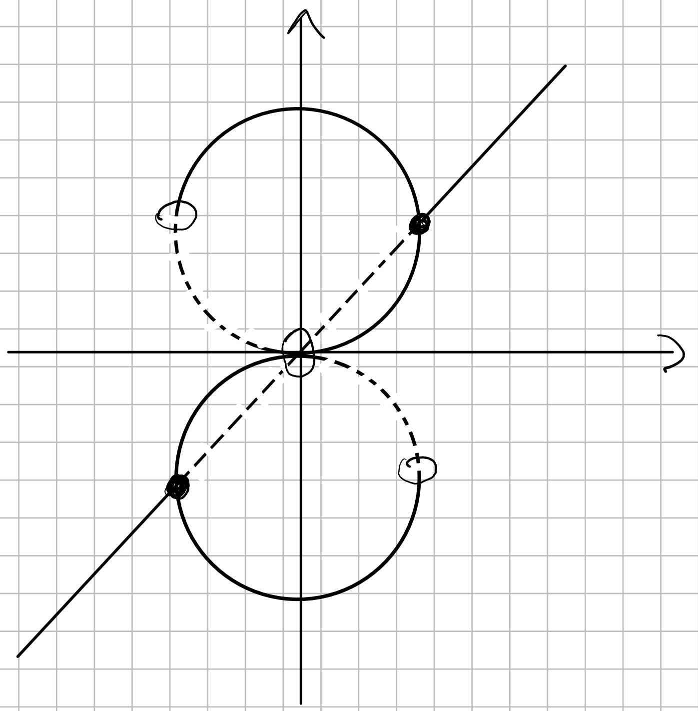
5/7/2024



n. 1

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n. 2 $\qquad x_{h+1} = \dfrac{k + x_h^2}{k + x_h}$

Equilibri: $\dfrac{k + x^2}{k + x} = x \Rightarrow k + \cancel{x^2} = kx + \cancel{x^2}$

$x \neq -k$

$kx = k \qquad k \neq 0 \Rightarrow x = 1 \quad (k \neq -1)$

$k = 0 \Rightarrow \forall x \neq 0$

Quindi:

$k = -1 \qquad \nexists$ equilibri

$k = 0 \qquad \forall \bar{x} \neq 0$

$k \neq 0, k \neq -1 \qquad \bar{x} = 1$

Stabilità: $f(x) = \dfrac{k + x^2}{k + x}$

$$f'(x) = \frac{2x(k+x) - (k+x^2)}{(k+x)^2} = \frac{2kx + 2x^2 - k - x^2}{(k+x)^2}$$

$$= \frac{x^2 + 2kx - k}{(k+x)^2}$$

$k \neq 0$
$k \neq -1$

$$f'(1) = \frac{\cancel{1+k}}{(1+k)^2} = \frac{1}{1+k}$$

$$\left| \frac{1}{1+k} \right| < 1 \quad \Rightarrow \quad |1+k| > 1$$

$$\Rightarrow \quad k < -2 \quad \vee \quad k > 0 \qquad \text{ASINT. STAB.}$$

$$-2 < k < 0 \qquad \text{INST.}$$

$k = -2$: $\quad f'(x) = \left. \frac{x^2 - 4x + 2}{(x-2)^2} \right|_{x=1} = -1$

$$f''(x) = \dots = \left. \frac{4}{(x-2)^3} \right|_{x=1} = 4 \qquad f'''(x) = \dots = \frac{-12}{(x-2)^4}$$

$$f'''(1) = -12$$

$2f'''(1) + 3f''(1)^2 = -24 + 48 > 0 \qquad x = 1$ A.S. per $k = -2$

$k = 0 \qquad x_{h+1} = x_h \qquad \forall \bar{x}$ STABILE SEMPLICEMENTE

(equilibrio indifferente)

$k = -2 \qquad x_{h+1} = \frac{x_h^2 - 2}{x_h - 2} \qquad f(x) = \frac{x^2 - 2}{x - 2}$

$$f(f(x)) = \frac{\left(\frac{x^2-2}{x-2}\right)^2 - 2}{\frac{x^2-2}{x-2} - 2} = \frac{(x^2-2)^2 - 2(x-2)^2}{(x-2)(x^2 - 2 - 2x + 4)} =$$

$$= \frac{x^4 - 6x^2 + 8x - 4}{x^3 - 4x^2 + 6x - 4} = x$$

$f(f(x)) = x \Rightarrow$

$$\cancel{x^4} - 6x^2 + 8x - 4 = \cancel{x^4} - 4x^3 + 6x^2 - 4x$$

$$4x^3 - 12x^2 + 12x - 4 = 0$$

$$x^3 - 3x^2 + 3x - 1 = 0$$

$$(x-1)^3 = 0 \Rightarrow x = 1$$

quindi non c'è 2-ciclo. $\square$

3) $\begin{cases} \dot{x} = -(k-1)x - y^3 \\ \dot{y} = (k-1)x - ky^3 \end{cases}$ $(0,0)$ $k \in \mathbb{R}$

$$\left.\begin{bmatrix} -(k-1) & 3y^2 \\ k-1 & -3ky^2 \end{bmatrix}\right|_{(0,0)} = \begin{bmatrix} -(k-1) & 0 \\ k-1 & 0 \end{bmatrix} \qquad \lambda = \begin{cases} -(k-1) \\ 0 \end{cases}$$

$-(k-1) > 0 \Rightarrow k-1 < 0 \Rightarrow k < 1$ $(0,0)$ INST.

$k \geq 1$

$k > 1$: $W(x,y) = \frac{x^2}{2}(k-1) + \frac{y^4}{4}$

$\dot{W} = -(k-1)^2 x^2 - ky^6 \leq 0$ $\{\dot{W} = 0\} = \{(0,0)\}$

$\Rightarrow (0,0)$ A. S. per $k > 1$

$k = 1$ $\begin{cases} \dot{x} = -y^3 \\ \dot{y} = -y^3 \end{cases}$ $\frac{d}{dt}(y-x) = 0 \Rightarrow x(t) = y(t) + c$

0 è stabile per $\dot{y} = -y^3$ $\Rightarrow$ $x(t) \to c$ $t \to +\infty$

$\Rightarrow$ $(0,0)$ stabile SEMPLICEMENTE per $k = 1$.

4) $\begin{cases} \dot{x} = \alpha x + y \\ \dot{y} = (1-\alpha)x + y \end{cases}$ $\begin{bmatrix} \alpha & 1 \\ 1-\alpha & 1 \end{bmatrix}$ $\tau = \alpha + 1$

$\delta = \alpha - (1-\alpha) = 2\alpha - 1$

$\delta < 0$ $\alpha < \frac{1}{2}$ SELLA

$\tau^2 - 4\delta = \alpha^2 + 2\alpha + 1 - 8\alpha + 4 = \alpha^2 - 6\alpha + 5$ $\alpha_{1/2} = 3 \pm \sqrt{9-5} = \begin{cases} 1 \\ 5 \end{cases}$

$\begin{cases} \alpha > \frac{1}{2} \\ 1 < \alpha < 5 \end{cases}$ $\Rightarrow$ $1 < \alpha < 5$ FUOCO INST. $\begin{cases} \tau < 0 & \alpha < -1 & \text{MAI} \\ \tau > 0 & \alpha > -1 & \forall \end{cases}$

$\frac{1}{2} < \alpha \leq 1$ $\vee$ $\alpha \geq 5$ NODO INST.

$\alpha = \frac{1}{2}$ ∞ pos. eq. $\lambda^2 - \frac{3}{2}\lambda \Rightarrow \begin{cases} \lambda = 0 \\ \lambda = \frac{3}{2} \end{cases}$ INST.