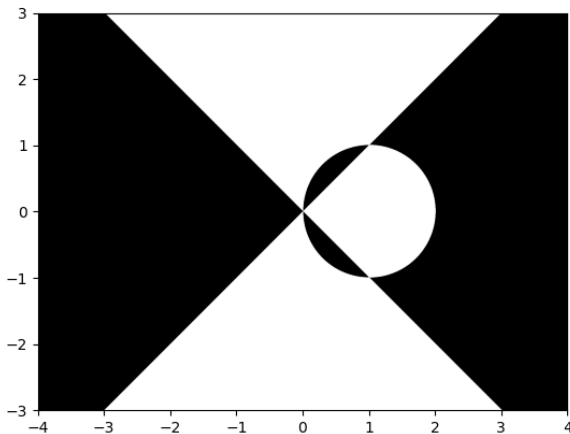


SD 7/2/25

1)



$$2) \begin{cases} \dot{x} = -x^3 - 2y \\ \dot{y} = \sin(2x) - ky \end{cases} \quad k \in \mathbb{R} \quad (0,0)$$

$$\begin{bmatrix} -3x^2 & -2 \\ 2\cos 2x & -k \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 2 & -k \end{bmatrix}$$

$$\lambda(k + \lambda) + 4 = 0 \quad \lambda^2 + k\lambda + 4 = 0$$

$$k < 0 \quad \text{UNST.}$$

$$k > 0 \quad \text{A.S.}$$

$$k=0 \quad \lambda = \pm 2i$$

$$\text{Ljapunov} \quad W = \int \sin 2x \, dx + y^2$$

$$\dot{W} = -x^3 \sin 2x - \cancel{2y \sin 2x} + \cancel{2y \sin 2x} \leq 0$$

$$\{ \dot{W} = 0 \} \Rightarrow x=0 \Rightarrow \dot{y}=0 \Rightarrow y=c$$

$$\dot{x}=0 \Rightarrow y=0 \quad \text{A.S.}$$

□

$$3) \begin{cases} \dot{x} = \alpha x + y \\ \dot{y} = (1-\alpha)x + y \end{cases} \quad \begin{bmatrix} \alpha & 1 \\ 1-\alpha & 1 \end{bmatrix}$$

$$r = \alpha + 1$$

$$\delta = \alpha - 1 + \alpha = 2\alpha - 1$$

$$2\alpha < 1$$

$$\alpha < \frac{1}{2}$$

SECCA

$$\alpha > \frac{1}{2}$$

$$\Rightarrow r > 0$$

INST.

$$r^2 - 4\delta = (\alpha+1)^2 - 8\alpha + 4$$

$$= \alpha^2 - 6\alpha + 5$$

$$r^2 - 4\delta \geq 0$$

per

$$\alpha \leq 1 \vee \alpha \geq 5$$

$$\frac{1}{2}$$

$$\frac{1}{2} < \alpha \leq 1 \vee \alpha \geq 5$$

NODO INST.

$$1 < \alpha < 5$$

FUOCO

INST.

4)

$$\begin{cases} x_{h+1} = \frac{1}{2}x_h + ky_h \\ y_{h+1} = -x_h - \frac{1}{2}y_h \end{cases}$$

$$k \in \mathbb{R}$$

$$\begin{cases} \frac{1}{2}x + ky = x \\ -x - \frac{1}{2}y = y \end{cases}$$

$$\begin{cases} \frac{1}{2}x = ky \\ x = -\frac{3}{2}y \end{cases}$$

$$-\frac{3}{4}y = ky$$

$$\left(k + \frac{3}{4}\right)y = 0$$

$$k \neq -\frac{3}{4}$$

$$y = 0$$

$$x = 0$$

$$(0, 0)$$

$$k = -\frac{3}{4}$$

$$\bar{y}$$

$$\bar{x} = -\frac{3}{2}\bar{y}$$

$$\left(-\frac{3}{2}\bar{y}, \bar{y}\right)$$

$$\begin{bmatrix} \frac{1}{2} & k \\ -1 & -\frac{1}{2} \end{bmatrix}$$

$$\lambda^2 + k - \frac{1}{4}$$

$$k > \frac{1}{4} \quad \lambda = \pm i \sqrt{k - \frac{1}{4}}$$

$$0 < k - \frac{1}{4} < 1 \Rightarrow k < \frac{5}{4} \text{ A.S.T.}$$

$$k > \frac{5}{4} \text{ INST}$$

$$k = \frac{5}{4} \text{ SEMPL. ST.}$$

$$k < \frac{1}{4} \quad \lambda^2 = \frac{1}{4} - k \quad \lambda = \pm \sqrt{\frac{1}{4} - k}$$

$$0 < \frac{1}{4} - k < 1 \quad k > -\frac{3}{4}$$

$$-\frac{3}{4} < k < \frac{1}{4} \text{ A.S.T.}$$

$$k < -\frac{3}{4} \text{ INST}$$

$$k = -\frac{3}{4} \text{ SEMPL. ST.}$$

$$k = \frac{1}{4}$$

$$\lambda^2 = 0$$

$$\begin{cases} \frac{1}{2}x + \frac{1}{4}y = x \\ -x - \frac{1}{2}y = y \end{cases}$$

$$\frac{1}{4}y = \frac{1}{2}x$$

$$-x = \frac{3}{2}y$$

$$(0, 0) \text{ A.S.}$$

