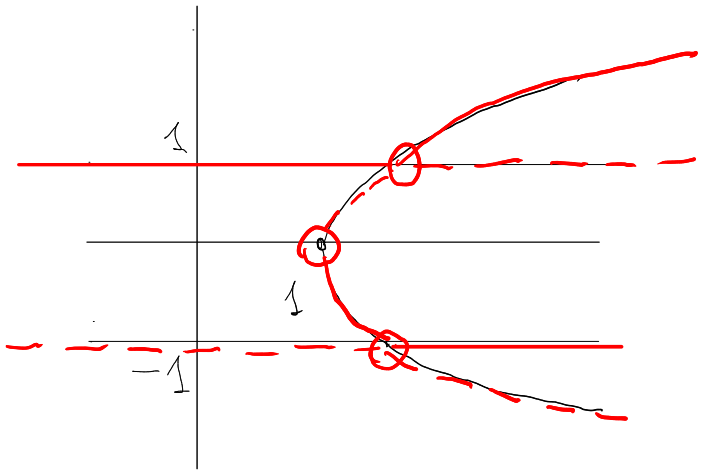
6/6/25 SD

1)



2)

$$\begin{bmatrix} -5x^4 & -3ky^2 \\ \cos x & -k \end{bmatrix}_{(0,0)} = \begin{bmatrix} 0 & 0 \\ 1 & -k \end{bmatrix}$$

$k<0$ INST.

$k>0 \quad W(x,y) = 1-\cos x + k\frac{y^4}{4}$

$$\dot{W} = -x^5\sin x - ky^3\sin x + ky^3\sin x - k^2y^4 \leq 0$$

$\{\dot{W}=0\} = \{(0,0)\} \Rightarrow (0,0)$ A.S.

Bacino di attrazione: $]-2\pi, 2\pi[\times \mathbb{R}$

$k=0$ $\begin{cases} \dot{x} = -x^5 & x=0 \text{ stabile} \quad f(x)\cdot(x-\bar{x})<0 \\ \dot{y} = \sin x & (0,\bar{y}) \text{ punti eq.} \end{cases}$

$\Rightarrow (0,\bar{y})$ sempl. stabili.

3) $\begin{bmatrix} \alpha & -\alpha \\ 2 & -1 \end{bmatrix}$ $\tau = \alpha - 1$ $\delta = -\alpha + 2\alpha = \alpha$

$\alpha < 0$ SELLE

$\alpha > 0$ $\tau^2 - 4\delta = \alpha^2 - 2\alpha + 1 - 4\alpha = \alpha^2 - 6\alpha + 1$

$\frac{\Delta}{4} = 9 - 1 = 8$ $\alpha_{1,2} = 3 \pm 2\sqrt{2}$

$0 < \alpha \leq 3 - 2\sqrt{2}$ $\alpha \geq 3 + 2\sqrt{2}$

NODO ST NODO INST

$3 - 2\sqrt{2} < \alpha < 1$ $1 < \alpha < 3 + 2\sqrt{2}$

FUOCO ST FUOCO INST.

$\alpha = 1$ $\tau = 0$ $\delta = 1 > 0$ CENTRO

$\alpha = 0$ ∞ per eq. $\begin{bmatrix} 0 & 0 \\ 2 & -1 \end{bmatrix}$ S. STAB.

4) $x_{k+1} = 2\alpha x_k - x_k^2$

$2\alpha x - x^2 = x$ $x^2 + (1 - 2\alpha)x = 0$

$x = 0$ $x = 2\alpha - 1$

$f'(x) = 2\alpha - 2x$ $f''(x) = -2$ $f'''(x) = 0$

$f'(0) = 2\alpha$ ST per $|\alpha| < \frac{1}{2}$

INST per $|\alpha| > \frac{1}{2}$

$\alpha = \frac{1}{2}$ $f'(0) = 1$ $f''(0) \neq 0$ INST

$\alpha = -\frac{1}{2}$ $f'(0) = -1$ $f''(0) = -2$ $f'''(0) = 0$

$2f''' + 3f''^2 > 0$ A.S.

$f'(2\alpha - 1) = 2\alpha - 2(2\alpha - 1) = 2\alpha - 4\alpha + 2 = -2\alpha + 2$

$-1 < 2\alpha - 2 < 1 \qquad 1 < 2\alpha < 3$

$\frac{1}{2} < \alpha < \frac{3}{2}$ A.S.

$\alpha < \frac{1}{2} \vee \alpha > \frac{3}{2}$ INST.

$\alpha = \frac{1}{2}$ $\bar{x} \equiv 0$ INST

$\alpha = \frac{3}{2}$ $\bar{x} = 2$ $f'(2) = -1$ $f'' \neq 0$ $f''' = 0$

A.S.

$2(2x - \alpha x^2) - \alpha(2x - \alpha x^2)^2 = x$

$\alpha = 1 \quad 4x - 2x^2 - (4x^2 + x^4 - 4x^3) = x$

$4 - 2x - 4x - x^3 + 4x^2 = 1$

$$x^3 - 4x^2 + 6x - 3 = 0$$

	1	−4	6	−3
1		1	−3	3
	1	−3	3	//

$x^2 - 3x + 3 \neq 0 \Rightarrow$ NO 2-ciclí.