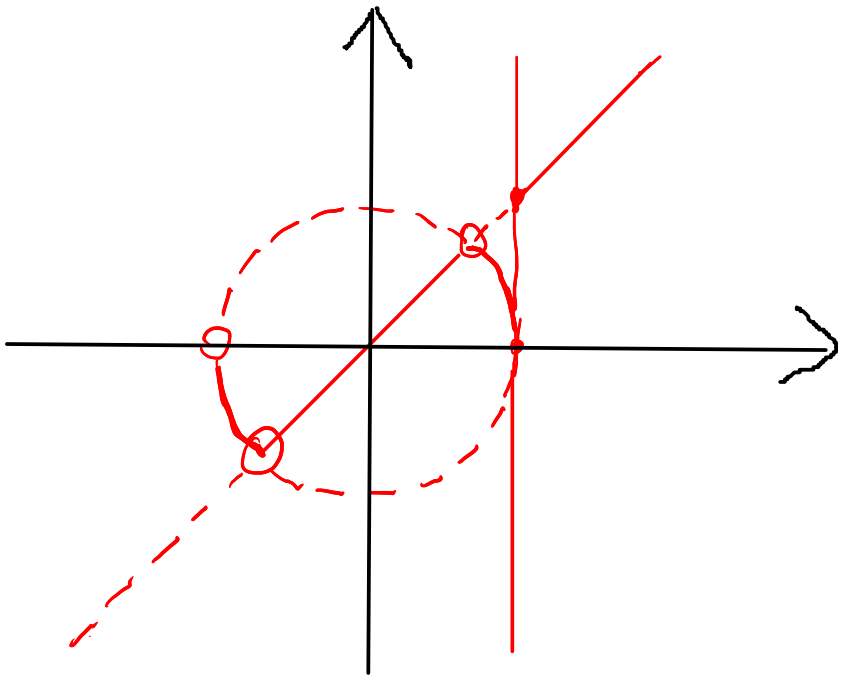
SD 20/6/2025

1)



2) $(0,0)$ $(k,2)$

$$JF = \begin{bmatrix} 4-2y & -2x \\ 2y & 2x-2k \end{bmatrix}$$

$$JF(0,0) = \begin{bmatrix} 4 & 0 \\ 0 & -2k \end{bmatrix}$$ INST. $\forall k$

$$JF(k,2) = \begin{bmatrix} 0 & -2k \\ 4 & 0 \end{bmatrix}$$ $\det = 8k$

Se $k<0$ INST (nella)

Se $k>0$ sempl. stabile (Lotka-Volterra)

Se $k=0$

$$\begin{cases} \dot{x} = x(4-2y) \\ \dot{y} = 2xy \end{cases}$$

$(0, \bar{y})$ eq. $\forall \bar{y}$

$$\begin{bmatrix} 4-2\bar{y} & 0 \\ 2\bar{y} & 0 \end{bmatrix}$$

$\bar{y} < 2$ INST.

$\bar{y} \geq 2$??

3) $A = \begin{bmatrix} \alpha & -1 \\ \alpha & 1 \end{bmatrix}$ $\tau = \alpha + 1$

$\delta = \alpha + \alpha = 2\alpha$

$\Delta = \tau^2 - 4\delta = \alpha^2 + 2\alpha + 1 - 8\alpha = \alpha^2 - 6\alpha + 1$

$\Delta \geq 0$ $\alpha_{1/2} = 3 \pm \sqrt{9-1} = 3 \pm 2\sqrt{2}$

$\alpha < 0$ STELLE

$0 < \alpha \leq 3 - 2\sqrt{2}$ $\vee$ $\alpha \geq 3 + 2\sqrt{2}$ NODI INST.

$3 - 2\sqrt{2} < \alpha < 3 + 2\sqrt{2}$ FUOCHI INST.

$\alpha = 0$ $\begin{bmatrix} 0 & -1 \\ 0 & 1 \end{bmatrix}$ INST.

$(\bar{x}, 0)$ eq.
$\forall \bar{x} \in \mathbb{R}$

4) $\alpha x(1+x^2) = x$

$x = 0 \qquad \alpha(1+x^2) = 1 \qquad \alpha \neq 0$

$1+x^2 = \frac{1}{\alpha}$

$x = \pm\sqrt{\frac{1}{\alpha} - 1}$ per $\frac{1}{\alpha} > 1$

$\alpha > 0 \Rightarrow \alpha < 1$ $\qquad 0 < \alpha < 1$

$\alpha < 0 \Rightarrow \alpha > 1$ NO

$f'(x) = \alpha + 3\alpha x^2 \qquad f''(x) = 6\alpha x \qquad f'''(x) = 6\alpha$

$f'(0) = \alpha$ $\qquad$ 0 A.S. per $-1 < \alpha < 1$

INST. per $\alpha < -1 \vee \alpha > 1$

$\alpha = 1 \qquad f'(0) = 1 \qquad f''(0) = 0 \qquad f'''(0) = 6 > 0$

$\Rightarrow$ 0 INST. per $\alpha = 1$

$\alpha = -1 \qquad f''(0) = 0 \qquad f'''(0) = -6$

$2f'''(0) + 3f''(0)^2 < 0$ INST.

$0 < \alpha < 1 \qquad f'\left(\pm\sqrt{\frac{1}{\alpha} - 1}\right) = \alpha + 3\alpha\left(\frac{1}{\alpha} - 1\right) =$

$= \alpha + 3 - 3\alpha = 3 - 2\alpha$

$-1 < 2\alpha - 3 < 1 \qquad 1 < \alpha < 2$ mai

$\Rightarrow \pm\sqrt{\frac{1}{2}-1}$ sempre instabile $(0<\alpha<1)$.

2-ciclo:

$$f^2(x) = x(1+x^2)\left(1+x^2(1+x^2)^2\right) = x$$

$$(1+x^2)\left(1+x^2(1+x^2)^2\right) = 1 \qquad x\neq 0$$

$$(1+x^2)(1+x^2+2x^4+x^6) = 1$$

$$\cancel{1}+x^2+2x^4+x^6+x^2+x^4+2x^6+x^8 = \cancel{1}$$

$$2x^2+3x^4+3x^6+x^8 = 0$$

$x\neq 0$ $\qquad 2+3x^2+3x^4+x^6 = 0$

$$1+(1+x^2)^3 = 0 \qquad \nexists\, x$$

$\nexists$ 2-ciclo $\square$