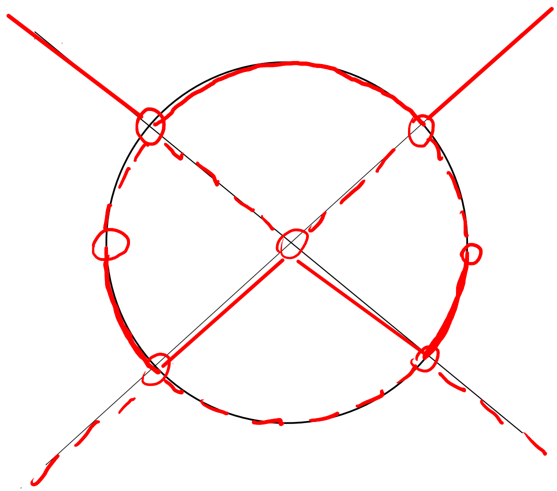
SD 18/7/25

1)



2)

$$\begin{cases} \dot{x} = x(k-y) \\ \dot{y} = y(2x-2) \end{cases} \quad k \neq 0 \qquad \begin{matrix} (0,0) \\ (1,k) \end{matrix}$$

$$H = \begin{bmatrix} k-y & -x \\ 2y & 2x-2 \end{bmatrix} \qquad (0,0) \quad \begin{bmatrix} k & 0 \\ 0 & -2 \end{bmatrix}$$

$k>0$ INST SELLA

$k<0$ A. ST. NODO

$(1,k)$ $\begin{bmatrix} 0 & -1 \\ 2k & 0 \end{bmatrix}$ $\delta = 2k$

$k<0$ INST (SELLA)

$k>0$ S. STAB. (Lotka – Volterra)

3) $\begin{cases} \dot{x} = \alpha \sin x - y \\ \dot{y} = \alpha x + y \end{cases}$

$y = -\alpha x \qquad \alpha \sin x + \alpha x = 0$

$\alpha \neq 0 \qquad \sin x + x = 0 \qquad x = 0$

$y = 0$

$\alpha = 0 \qquad (\bar{x}, 0)$

$$\begin{bmatrix} \alpha \cos x & -1 \\ \alpha & 1 \end{bmatrix} \qquad (0,0) \quad \begin{bmatrix} \alpha & -1 \\ \alpha & 1 \end{bmatrix}$$

$\alpha \neq 0 \qquad \tau = \alpha + 1 \qquad \delta = 2\alpha \neq 0$

$\alpha < 0$ SELLA

$\alpha > 0 \qquad \tau^2 - 4\delta = \alpha^2 + 2\alpha + 1 - 8\alpha$

$= \alpha^2 - 6\alpha + 1$

$\alpha_{1/2} = 3 \pm \sqrt{8}$

$0 < \alpha \leq 3 - \sqrt{8} \quad \vee \quad \alpha \geq 3 + \sqrt{8}$ NODO INST.

$3 - \sqrt{8} < \alpha < 3 + \sqrt{8}$ FUOCO INST

$\alpha = 0 \qquad \begin{cases} \dot{x} = -y \\ \dot{y} = y \end{cases} \qquad \begin{bmatrix} 0 & -1 \\ 0 & 1 \end{bmatrix}$ INST.

4) $$\frac{\alpha x}{1+2x+x^2} = x$$

$x = 0$ $\qquad (1+x)^2 = \alpha$ $\qquad \alpha \geq 0$

$$1+x = \pm\sqrt{\alpha}$$

$$x = -1 \pm \sqrt{\alpha}$$

$$f'(x) = \frac{\alpha(1+x)^2 - 2(1+x)\alpha x}{(1+x)^4} = \frac{\alpha(1+x) - 2\alpha x}{(1+x)^3}$$

$$= \frac{\alpha + \alpha x - 2\alpha x}{(1+x)^3} = \frac{\alpha(1-x)}{(1+x)^3}$$

$f'(0) = \alpha$ $\qquad |\alpha| < 1$ A.S.

$\qquad |\alpha| > 1$ INST

$$f''(x) = \frac{-\alpha(1+x)^3 - 3(1+x)^2\alpha(1-x)}{(1+x)^6}$$

$$= \frac{-\alpha(1+x) - 3\alpha(1-x)}{(1+x)^4}$$

$$= \frac{-\alpha(1+x+3-3x)}{(1+x)^4} = \frac{-2\alpha(2-x)}{(1+x)^4}$$

$\alpha = 1$

$$f''(0) = -2\alpha = -2 \neq 0 \qquad \text{IN ST.}$$

$\alpha = -1$

$$f'''(x) = \frac{2\alpha(1+x)^4 + 2\alpha(2-x)4(1+x)^3}{(1+x)^8}$$

$$= \frac{2\alpha(1+x) + 8\alpha(2-x)}{(1+x)^5} =$$

$$= \frac{2\alpha(1+x+8-4x)}{''} = \frac{2\alpha(9-3x)}{(1+x)^5}$$

$$f'''(0) = 18\alpha = -18 \qquad f''(0) = -4\alpha = 4$$

$$2f'''(0) + 3f''(0)^2 = -36 + 48 > 0 \qquad \text{ST}$$

$\alpha > 0$

$$f'(-1 \pm \sqrt{\alpha}) = \frac{\cancel{\alpha}(1-x)}{\cancel{\alpha}\sqrt{\alpha}} = \frac{1+1 \mp \sqrt{\alpha}}{\sqrt{\alpha}} = \frac{2 \mp \sqrt{\alpha}}{\sqrt{\alpha}}$$

$$-1 < \frac{2+\sqrt{\alpha}}{\sqrt{\alpha}} < 1 \qquad 2+\sqrt{\alpha} < \sqrt{\alpha} \qquad \text{MAI}$$

$$\Rightarrow \qquad -1-\sqrt{\alpha} \qquad \text{INST}$$

$$-1 < \frac{2-\sqrt{\alpha}}{\sqrt{\alpha}} < 1 \qquad -\sqrt{\alpha} < \sqrt{\alpha}-2 < \sqrt{\alpha}$$

$$\checkmark$$

$$2\sqrt{\alpha} > 2 \qquad \sqrt{\alpha} > 1 \qquad \alpha > 1$$

A. S.

$\alpha < 1$ INST.

$2 \doteq 1$ $\qquad -1+\sqrt{2} = 0$ $\qquad$ IN ST.

$\square$