SD 5/9/25

1) $\begin{cases} \dot{x} = x(k-y) \\ \dot{y} = y(kx-1) \end{cases}$ $k \neq 0$ $(0,0)$ $\left(\frac{1}{k}, k\right)$

$$\begin{bmatrix} k-y & -x \\ ky & kx-1 \end{bmatrix}$$

$(0,0)$ $\begin{bmatrix} k & 0 \\ 0 & -1 \end{bmatrix}$ $k>0$ INST. sella

$k<0$ A.S. nodo

$\left(\frac{1}{k}, k\right)$ $\begin{bmatrix} 0 & -\frac{1}{k} \\ k^2 & 0 \end{bmatrix}$ $\lambda^2 + k = 0$

$k<0$ INST sella

$k>0$ L-V

$W = kx - \ln x + y - k \ln y$ $\dot{W} = 0$

$\Rightarrow \left(\frac{1}{k}, k\right)$ centro per $k>0$. □

2) $x_{n+1} = x_n + \frac{\alpha x_n^2}{1+x_n^2}$

$\frac{\alpha x^2}{1+x^2} = 0$ $\alpha \neq 0 \Rightarrow x = 0$

$\alpha = 0 \Rightarrow \forall x$ EQ. INDIFF.

$\alpha \neq 0$

$$f'(x) = 1 + \frac{2\alpha x(1+x^2) - \alpha x^2 \cdot 2x}{(1+x^2)^2}$$

$$= 1 + \frac{2\alpha x + 2\alpha x^3 - 2\alpha x^3}{(1+x^2)^2} = 1 + \frac{2\alpha x}{(1+x^2)^2}$$

$$f'(0) = 1$$

$$f''(x) = \frac{2\alpha(1+x^2)^2 - 2\alpha x \cdot 2(1+x^2)2x}{(1+x^2)^4}$$

$$= \frac{(1+x^2)\left[2\alpha(1+x^2) - 8\alpha x^2\right]}{(1+x^2)^4}$$

$$= \frac{2\alpha - 6\alpha x^2}{(1+x^2)^3}$$

$$f''(0) = 2\alpha \neq 0 \Rightarrow 0 \text{ INST.} \quad \square$$

3)
$$\begin{cases} \dot{x} = -\sin x \\ \dot{y} = -\alpha y - \alpha y z \\ \dot{z} = \alpha y^2 - z^3 \end{cases}$$

$\alpha \in \mathbb{R}$

$(0,0,0)$

$$\begin{bmatrix} -\cos x & 0 & 0 \\ 0 & -\alpha - \alpha z & -\alpha y \\ 0 & 2\alpha y & -3z^2 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\alpha & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\alpha < 0$ INST $\qquad \alpha \geq 0$?

Ljapunov:

$$\begin{cases} \dot{x} = -\sin x & \cdot \sin x \\ \dot{y} = -\alpha y - \alpha y z & \cdot y \\ \dot{z} = \alpha y^2 - z^3 & \cdot z \end{cases}$$

$$W = 1 - \cos x + \frac{y^2}{2} + \frac{z^2}{2}$$

$$\dot{W} = -\sin^2 x - \alpha y^2 - \cancel{\alpha y^2 z} + \cancel{\alpha y^2 z} - z^4 \leq 0$$

$\alpha > 0$ $\quad \{\dot{W} = 0\} = \{(0,0,0)\}$ A.S.

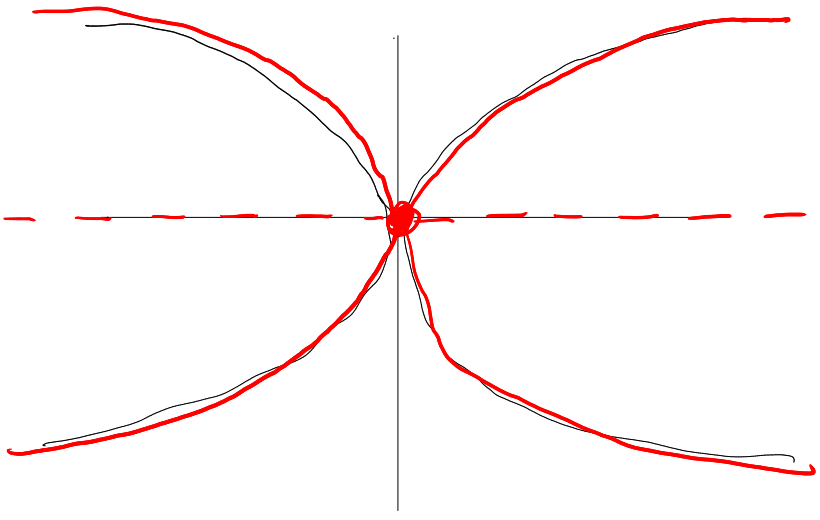
$\alpha = 0$ $\quad \{\dot{W} = 0\} = \{-\sin^2 x - z^4 = 0\} = \{(0, y, 0)\}$

$$\begin{cases} \dot{x} = -\sin x \\ \dot{y} = 0 \\ \dot{z} = -z^3 \end{cases}$$

$(0, \bar{y}, 0)$ ∞ pos. eq.

STAB. SEMPL. □

4) $\dot{x} = x(|\mu| - x^2)$



$\mu = 0 \qquad \dot{x} = -x^3 \qquad \dot{x}\, x^{-3} = -1$

$$\frac{x^{-2}}{-2} = -t + c$$

$$\frac{1}{x^2} = 2t + c \qquad x^2 = \frac{1}{2t+c}$$

$$x = \pm \frac{1}{\sqrt{2t+c}}$$

$x(0) = 1 \Rightarrow \pm \frac{1}{\sqrt{c}} = 1 \Rightarrow c = 1$

scelgo +

$$\Rightarrow x(t) = \frac{1}{\sqrt{2t+1}}$$

□